The Logical Structure of Phonological Generalizations

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Today

1. I will argue that the logical structure of phonological generalizations is

   1. not only "regular", but also
   2. "less than" regular in a particularly "local" way.

2. These concepts provide a way to understand

   1. the extensive variation cross-linguistically, and
   2. how these patterns can be acquired from examples, and
   3. the important role played by representation in grammar.

Today

3. Formalizing these insights directly with logic provides a better theory of phonology than theories

1. based on global optimization like Optimality Theory, or
2. based on serial rule application.

Requires two books:

- "encyclopedia of categories"
- "encyclopedia of types"

Wilhelm Von Humboldt

An Encyclopedia of Categories: Logic and Representation

Logic

- Monadic Second Order
- First Order
- Propositional
- Conjunctions of Positive and Negative Literals
- Other fragments of MSO logic
- ...

Representations

- Segments
- Binary Features
- Scalar Features (aperture)
- Elements
- Syllabic Roles, Metrical Feet, Autosegments
- Articulatory Scores, Coupling Graphs
- ...

Outline

Part I

What is phonology?

The fundamental insight

The fundamental insight in the 20th century which shaped the development of generative phonology is the following.

The **best** explanation of the systematic variation in the pronunciation of morphemes is to posit a single underlying mental representation of the phonetic form of each morpheme and to derive its pronounced variants with context-sensitive transformations.

Arguments are given in Kenstowicz and Kisseberth 1979, chap 6 and Odden 2014, chap 5)

Example from Finnish

Nominative Singular	Partitive Singular	
aamu	aamua	‘morning’
kello	kelloa	‘clock’
kylmæ	kylmææ	‘cold’
kømpelø	kømpeløæ	‘clumsy’
æiti	æitiæ	‘mother’
tukki	tukkia	‘log’
yoki	yokea	‘river’
ovi	ovea	‘door’

Lexicon + Grammar

Lexicon

MOTHER	LOG	RIVER	DOOR
æiti	tukki	yoke	ove

Grammar realizing Word-final /e/ raising)

1. e $\longrightarrow$ [+high] / __ #
2. *e# >> Ident(high)

If your theory asserts that ...

There exist underlying representations of morphemes which are transformed to surface representations...

Then there are three important questions:

1. **What is the nature** of the abstract, underlying, lexical representations?
2. **What is the nature** of the concrete, surface representations?
3. **What is the nature** of the transformation from underlying forms to surface forms?

Theories of Phonology...

- disagree on the answers to these questions, but *they agree on the questions being asked.*

Extensions of grammars are infinite objects like circles.

Word-final /e/ raising | **Circle with radius 1**

Intensional Descriptions

Word-final /e/ raising

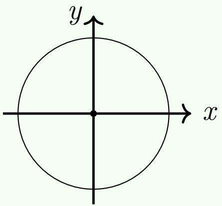
1. e ⟶ [+high] / __ #
2. *e# >> IDENT(HIGH)

Circle with radius 1

1. Cartesian: $x^2 + y^2 = 1$
2. Polar: $r = 1$

Extensional Descriptions

(ove,ovi), (yoke,yoki), ...
(tukki,tukki), (kello,kello), ...
(manilabanile,manilabanili), ...



Truisms about grammars

1. Different grammars may generate the same constraints and transformations just like different equations can realize the same functions.
2. Grammars may have properties *largely independent* of grammatical particulars.
    - regular sets and functions (Kleene 1956, Elgot and Mezei 1965, Scott and Rabin 1959)
    - output-driven maps (Tesar 2014)
    - strict locality (Rogers and Pullum 2011)

Part II

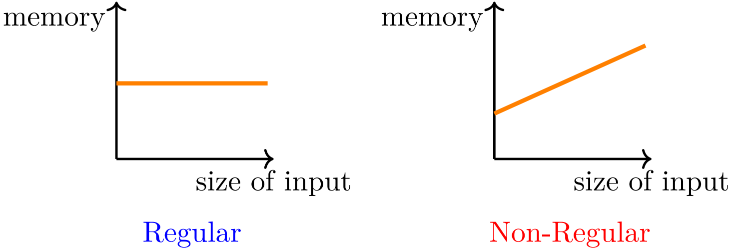
Phonological Generalizations are Regular

Regular grammars for constraints and transformations

1. Monadic Second Order (MSO) logic
2. Regular expressions
3. Finite-state automata

What "Regular" means

A grammar is regular provided **the memory required to assemble the computation is bounded by a constant, *regardless of the size of the input.***



Some computations important to grammar

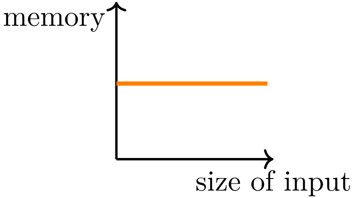
1. For given constraint C and any representation R:
   Does R violate C? How many times?
2. For given grammar G and any underlying representation R:
   What surface representation(s) does G transform R to?
   With what probabilities?

Example: Vowel Harmony

Progressive
Vowels agree in backness with the first vowel in the underlying representation.
Majority Rules
Vowels agree in backness with the majority of vowels in the underlying representation.

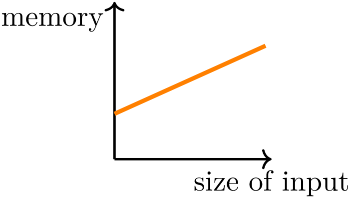
UR	Progressive	Majority Rules
/nokelu/	nokolu	nokolu
/nokeli/	nokolu	nikeli
/pidugo/	pidige	pudugo
/pidugomemi/	pidigememi	pidigememi

(Bakovic 2000, Finley 2008, 2011, Heinz and Lai 2013)



Regular

Progessive



Non-Regular

Majority Rules

Some Perspective

Typological

Majority Rules is unattested. (Bakovic 2000, Dolatian and Karakas, submitted)

Psychological

Human subjects fail to learn Majority Rules in artificial grammar learning experiments, unlike progressive harmony. (Finley 2008, 2011)

Computational

Majority Rules is not regular. (Riggle 2004, Heinz and Lai 2013)

Optimality Theory

1. There exists a CON and ranking over it which generates Majority Rules: Agree(back)>>IdentIO[back].

2. Some believe changing CON may resolve this, but such an approach fails to recognize the core problem.

3. The problem appears to be global optimization itself. There are many reasons to think CON cannot be so changed! (Hao 2019, 2024; Lamont 2021, 2022)

Phonological generalizations are regular

Evidence supporting the hypothesis that phonological generalizations are regular originate with Johnson (1972) and Kaplan and Kay (1994), who showed how to translate any ordered sequence of SPE-style rewrite rules into a finite-state automaton.

Consequently:

1. Constraints on well-formed surface and underlying representations are regular (since the image and pre-image of finite-state functions are finite-state, Rabin and Scott 1959)
2. Since virtually any phonological grammar can be expressed as an ordered sequence of SPE-style rewrite rules, this means "being regular" is a property of the functions that *any* phonological grammar defines.

Part III

Expressing Grammars with Logic

Word-final e-raising in Finnish implicates the constraint *e#.

Tips:

1. Think of variables like x and y as *positions* in a sequence.
2. $\texttt{property}(x)$ means position x has that property.
3. $x \triangleleft y$ means position y is the next position after x
4. $\wedge$ means AND; $\vee$ means OR; $\neg$ means NOT; $\exists$ means EXISTS

$$\texttt{e}(x) \stackrel{\text{def}}{=} \texttt{vocalic}(x) \wedge \texttt{mid}(x) \wedge \texttt{front}(x)$$

$$\texttt{final}(x) \stackrel{\text{def}}{=} (\exists y)[x \triangleleft y \wedge \#(y)]$$

$$*\texttt{e}\# \stackrel{\text{def}}{=} \neg(\exists x)[\texttt{e}(x) \wedge \texttt{final}(x)]$$

Illustrating structure

$$\texttt{e}(x) \stackrel{\text{def}}{=} \texttt{vocalic}(x) \wedge \texttt{mid}(x) \wedge \texttt{front}(x)$$

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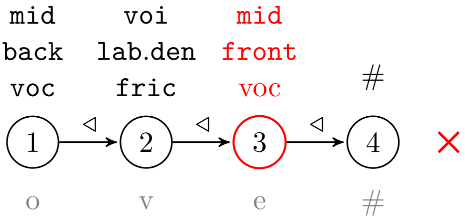

The relational structure of the input /ove/ ‘door, nom-sg.’

Illustrating structure

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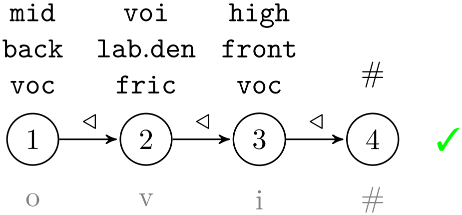


The relational structure of the output [ovi] ‘door, nom-sg.’

Transformations in First Order Logic

Logical expressions can translate one structure into another with a collection of sentences defining properties of the positions in the output.

$$\phi P(x) \quad \stackrel{\text{def}}{=} \quad Q(x)$$

"Position x has property P in the *output* only if corresponding position x in the *input* satsifies predicate Q."

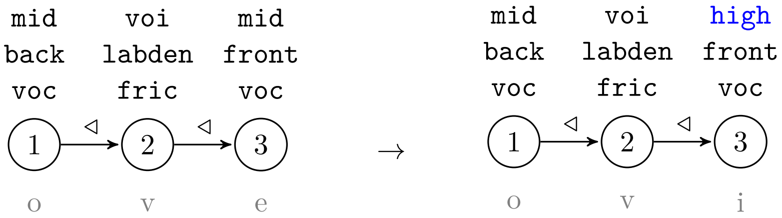
(Courcelle 1994, Engelfriedt and Higgeboom 2001)

Word Final /e/ raising in Finnish

$$\phi\texttt{high}(x) \overset{\text{def}}{=} \texttt{high}(x) \vee \big(\texttt{e}(x) \wedge \texttt{final}(x)\big)$$

$$\phi\texttt{mid}(x) \overset{\text{def}}{=} \texttt{mid}(x) \wedge \neg\big(\texttt{e}(x) \wedge \texttt{final}(x)\big)$$

$$\phi\texttt{feature}(x) \overset{\text{def}}{=} \texttt{feature}(x) \text{ (for all other features)}$$



The input /ove/ and output [ovi] ‘door, nom-sg.’

Logic: So What?

1. Choosing a logical formalism and a representation fixes a theory.
2. Any logic equivalent to some fragment of MSO is at most regular.
3. Weaker logics facilitate learning and acquisition

Consider Constraints.

- Constraints definable with Monadic Second Order (MSO) logic over strings are regular.
- But we only used First Order logic for Finnish.
- Do we need MSO logic?
- Can we go below FO logic?

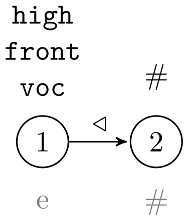
Part IV

The Strengths of Weaker Logics

Propositional Logic over Factors

1. Propositional logic has no variables like x nor quantifers like $\exists$. It only uses connectives like $\wedge, \vee$ and $\neg$.
2. Literals are connected pieces of structure called factors.
3. Structure S satisfies a factor F if it contains it, and so Structure S satisfies formula $\neg F$ if S does not contain F.

Define the factor e# as this structure:



Consider the formula ¬e#. The structure of /ove/ does not satisfy this formula because it *contains* that factor!

(Rogers and Lambert 2019)

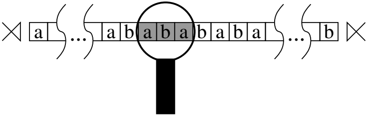
Conjunctions of postive and negative literals

Formulas of the form

$$\neg F_1 \wedge \neg F_2 \ldots \wedge F_k \ldots \wedge F_n$$

mean that well-formed structures:

do not contain factor F_1
and do not contain factor F_2
...
and contain factor F_k.
...
and contain factor F_n.



Factors like F_1 are forbidden and factors like F_n are required.

How much can phonological well-formedness be expressed with such a fragment of propositional logic?

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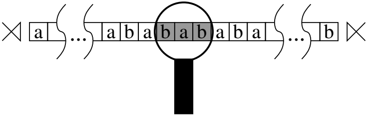

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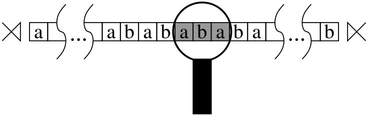

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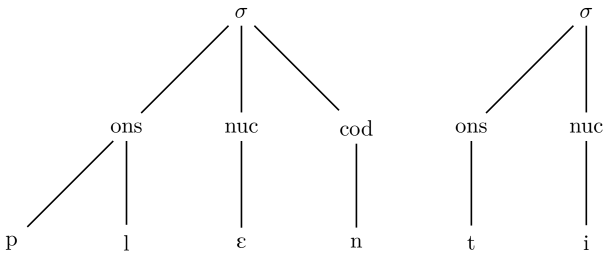
Non-lexical Stress Patterns

The StressTyp2 Database provides non-lexical stress patterns for over 400 languages. 106 distinct patterns have been encoded with finite-state machines. (Goedemans et al. 2015)

Rogers and Lambert (2019) show that:

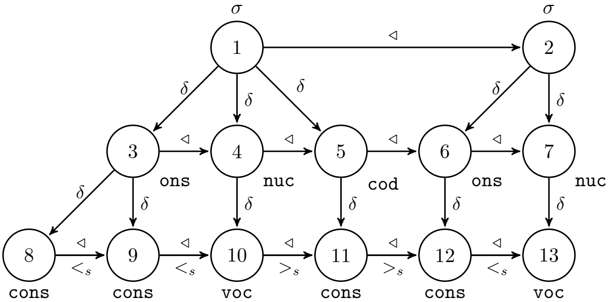
- 98/106 patterns can be described as the conjunction of positive and negative literals.
- 6 more require implication: "F_1 implies $\neg F_2$"
- 2 require MSO logic because they contain "hidden alternation pattern that requires an odd number of syllables to occur in certain spans of the word," and these become simple if secondary stress is perceptible.

Syllable Structure



(Strother-Garcia 2019)

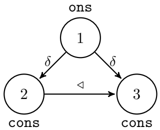
Syllable Structure



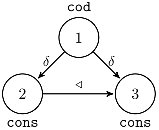
(Strother-Garcia 2019)

Syllable Structure

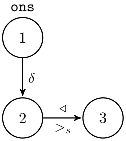
`ComplexOnset` $\stackrel{\text{def}}{=}$



`ComplexCoda` $\stackrel{\text{def}}{=}$



`OnsetSequencing` $\stackrel{\text{def}}{=}$



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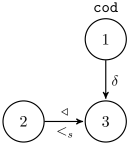

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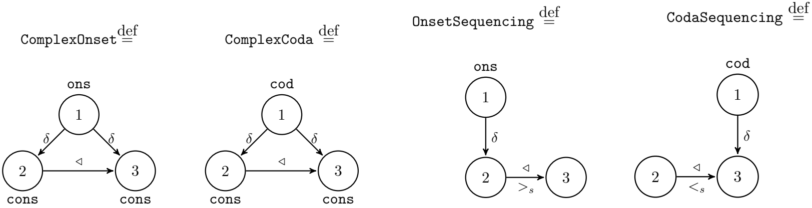


Strother-Garcia 2019 shows that:

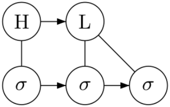
- Constraints given by Onset, NoCoda, and the Sonority Sequencing Principle follow from forbidding particular factors.
- Basic CV typology, and extensions thereof, are obtained from different combinations of these forbidden factors.

(Strother-Garcia 2019)

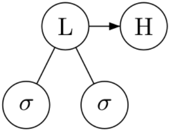
Tonal Mapping

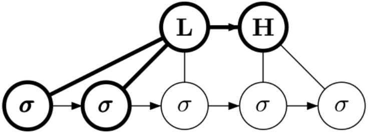
Autosegmental representations are also relational structures (Goldsmith 1976, Coleman and Local 1991).

félàmà HLL ‘junction’
(Mende)



Factors are connected pieces of structure





(Jardine 2016, 2017)

Tonal Mapping

Jardine 2016, 2017 shows that well-studied patterns of tonal association can all be expressed by forbidding the right factors.

1. Position-specific plateaus (Mende, Hausa, Northern Karanga)
2. Position-specific contours (Mende, Hausa, Northern Karanga)
3. Melody constraints (Mende)
4. Quality-dependent plateaus (Kukuya)

(Jardine 2016, 2017)

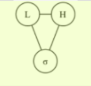
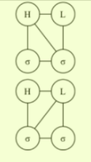
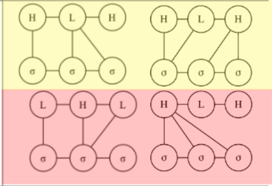
Learning Constraints

1. The examples on the previous slides were inviolable, language-specific constraints.
2. Chandlee et al. 2019 show that the space of factors form a partial order, and design the Bottom Up Factor Inference Algorithm (BUFIA) to search this space for forbidden factors.

(Chandlee et al. 2019, Payne 2024)

Result

664 native words (in orthography, with tonal markers) from a Hausa mini-dictionary were used as a positive data. BUFIA identified 7 banned structures when syllable and tone numbers are ≤ 3.

	$t=1$	$t=2$	$t=3$
$s=1$	Found nothing 0		Nothing New Found -
$s=2$	0		-
$s=3$	0	-	

Some have been reported before in linguistic analyses

Some are more specific than previously reported generalization

Some have never been discussed

(Li 2024)

BUFIA applied to Quechua

Examining local and long-distance phonotactics found in Quechua, Wilson & Gallagher (2018) argue phonotactic learning over featural representations necessitates statistical methods such as Maximum Entropy (Hayes and Wilson 2008).

Swanson et al. (submitted) show BUFIA performs well.

	held-out forms (W&G)	legal nonce roots	illegal nonce roots
Features, Stats (MaxEnt-Ftr)	99.8%	82.2%	1.9%
Segments, Stats (MaxEnt-Seg)	99.7%	71.5%	45.4%
Segments, No-stats ((T)SL)	96.7%	18.8%	0.1%
Features, No-stats (BUFIA)	99.6%	94.1%	1.8%

Percentage of forms accepted by evaluation category aggregated over the five folds. Results reported in rows 1-3 are from W&G.

Logics weaker than First Order have strengths!

1. Good typological coverage
2. Good theoretical and empirical learning algorithms

Also, algebraic methods exist which help us determine how logically complex a given constraint may be (Lambert 2022).

Interim Summary

Logics weaker than First Order have strengths!

1. Good typological coverage
2. Good theoretical and empirical learning algorithms

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Do weaker logics for transformations have the same strengths?

Quantifier Free (QF) Transformations

Like Propositional Logic, QF logic is weaker than FO logic.

Compare:

1. $\phi P(x) \stackrel{\text{def}}{=} Q(x) \wedge \exists y[R(y)]$ (First Order Definable)

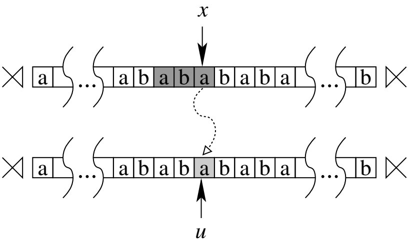
   Requires scanning whole word for such a y!!

2. $\phi P(x) \stackrel{\text{def}}{=} Q(x) \wedge R(x)$ (QF Definable)

   Information to decide P is local to x in the input!!

QF is Input Strictly Local

Lindell and Chandlee (2016) show that Quantifier-Free transductions over strings are Input Strictly Local (ISL) transformations, which are schematized here:



For every Input Strictly 3-Local function, the output string u of each input element x depends only on x and the 2 elements previous to x. In other words, the contents of the lightly shaded cell only depends on the contents of the darkly shaded cells.

Chandlee 2014, Chandlee and Heinz 2018

ISL transformations in phonology:

1. Approximately 95% of the individual processes in P-Base (v.1.95, Mielke 2008), including local substitution, deletion, epenthesis, and synchronic metathesis
2. Many *opaque* transformations without any special modification.

Many phonological patterns, including many opaque ones, have the necessary information to decide the output contained within windows of bounded length on the input side.

Chandlee 2014, Chandlee and Heinz 2018, Chandlee, Heinz and Jardine 2018

More Strengths of ISL transformations

1. Chandlee et al. 2014 show ISL transformations are learnable from positive examples given a bound on the window size.
2. They can be generalized to operate on a 'tier' to account for long-distance harmony and spreading processes (McMullin 2016, Burness and McMullin 2019, Burness et al 2021, Lambert and Heinz 2024)
3. Given an arbitrary finite-state transducer, one can decide whether it is (tier) ISL or not (Lambert and Heinz 2023).

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What about non-linear representations?

QF transformations with nonlinear representations

Strother-Garcia (2018) shows that the process of syllabification in Imdlawn Tashlhiyt Berber (Dell and Elmedlaoui, Prince and Smolensky 1993) is Quantifer Free. She concludes

"...syllabification in ITB can be represented by a QF [logical] transduction, a formalism restricted to substantially lower computational complexity than [traditional] phonological grammars...Establishing that ITB syllabification is QF highlights an insight not apparent from [those traditional] grammatical formalisms..."

QF transformations with nonlinear representations

Dolatian (2020) examines the phonology-morphology interface in light of Quantifier Free logical transductions. He concludes

"the bulk of the morphology-phonology interface requires local computation, not global computation."

Comparing and Translating Representations

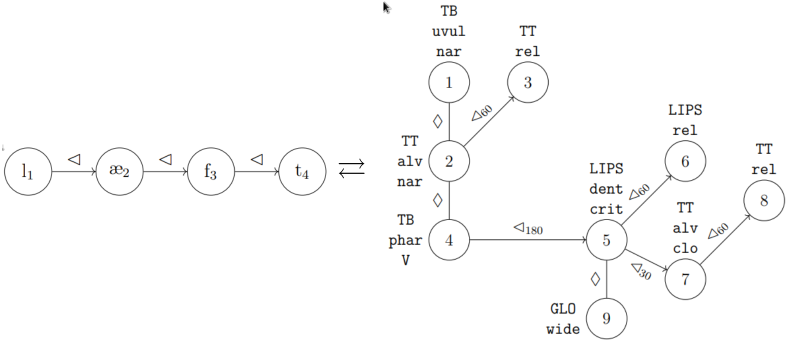
Oakden (2020) compares Bao's and Yips's tonal models and demonstrates their inter-translatability with quantifier free logic, concluding they are notational variants.

Comparing and Translating Representations

Jardine et al. (2021) compare autosegmental representations with Q-theory (Shih and Inkelas 2019) using quantifier free logic and shows that Q-theory is mostly the same as autosegmental representations contra their claims.

Comparing and Translating Representations

Nelson (2022) examines the phonetics-phonology interface. In particular he shows how to convert coupling graphs in Articulatory Phonology to familiar segmental representations and vice versa using first order logic.



Part V

Concluding Thoughts

Discussion

1. Logic and relational structures can express phonological generalizations precisely, accurately, and completely.
2. They provide an “Encyclopedia of categories” (Humboldt) and so can be used to compare phonological theories.
3. They inform models of representation, memory and processing.
4. They are easy to learn with only a little practice.
5. They can be *weighted* to compute probabilties, count violations, handle optionality, ...
6. Language-specific inviolable constraints are not a problem if they can be learned!
7. Weaker logics admit learnability results that more expressive logics cannot!
8. Logic will still be here in 100, 200 years...!!

Conclusion

The logical structure of phonology reveals

1. that much (all?) of phonology is local with the right representations,
2. that the computations are mostly (all?) subregular (below FO),
3. what learners must attend to, and thus explains the kinds of phonological generalizations that can be learned, and in this way provides
4. explanations for the phonological generalizations we do and do not observe.

Current and Future Work

1. Examine more representations, such as element theory/government phonology
2. Examine more logics including Boolean Monadic Recursive Schemes (Chandlee and Jardine 2021)
3. Learning lexicons, grammars, exceptions, variation
4. Learning transformations over non-linear representations

Finally, I am delighted to say that, with Chandlee and Jardine, we have just received a grant from the United States National Science Foundation to pursue precisely these last two items!

ありがとうございました！

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