

# Culminativity times Harmony Equals Unbounded Stress Patterns

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The 2nd Workshop on Stress and Accent  
University of Connecticut  
December 3, 2011

Culminativity  $\times$  Harmony = Unbounded Stress Patterns

$$\text{Harmony} = \frac{\text{Unbounded Stress Patterns}}{\text{Culminativity}}$$

## What are Bounded and Unbounded Stress Patterns?

1. Bounded stress patterns are ones where the primary stress always falls within some fixed distance of the word edge.
2. Unbounded patterns are not bounded.

## Unbounded Stress Patterns

Words obeying the stress pattern of Kwakiutl (Bach 1975)

H́	Ĺ	H́ L	H́ H	L H́
L Ĺ	H́ L L	H́ L H	H́ H L	H́ H H
L H́ L	L H́ H	L L Ĺ	L L H́	L H́ L L
L H́ L H	H́ L L L	H́ L L H	H́ H L L	H́ H L H
L H́ H L	L H́ H H	H́ L H L	H́ L H H	H́ H H L
H́ H H H	L L H́ L	L L H́ H	L L L Ĺ	L L L H́

## Unbounded Stress Patterns

Words obeying the stress pattern of Kwakiutl (Bach 1975)

́	́	́ L	́ H	L ́
L ́	́ L L	́ L H	́ H L	́ H H
L ́ L	L ́ H	L L ́	L L ́	L ́ L L
L ́ L H	́ L L L	́ L L H	́ H L L	́ H L H
L ́ H L	L ́ H H	́ L H L	́ L H H	́ H H L
́ H H H	L L ́ L	L L ́ H	L L L ́	L L L ́

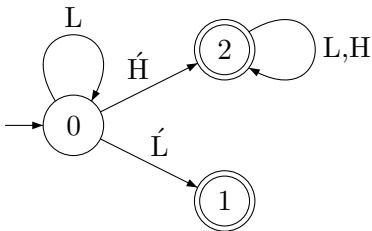
The generalization: “Stress the Leftmost Heavy otherwise the Rightmost” (LHOR)

## Typology of Unbounded Stress Patterns

1. LHOR (e.g. Kwakitul)
  2. LHOL (e.g. Amele)
  3. RHOR (e.g. Golin)
  4. RHOL (e.g. Chuvash)
- More complicated unbounded patterns have been documented
  - Typological studies document lexical exceptions to generalizations.
  - Here the focus is on the nature of the generalizations.

(Hyman 1977, Halle and Verganud 1987, Idsardi 1992, Bailey 1995, Hayes 1995, Goedemans et al. 1996, Tesar 1998, Gordon 2002, Heinz 2007, 2009, Hulst et al. 2010)

# LHOR generalization as a finite-state automaton



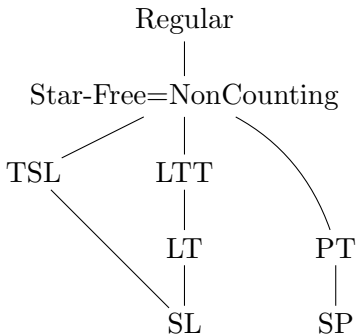
LHOR



## The nature of the generalization

1. The LHOR fsa computes an infinite set:
  - every word in the set obeys the generalization
  - every word not in the set does not obey the generalization
2. SPE and OT analyses of this generalization compute the *same* infinite set (the right projection of the UR/SR relation)
3. The focus in this talk is WHAT is being computed as opposed to HOW it is computed.

## Classifying regular patterns



Proper inclusion relationships among language classes (indicated from top to bottom).

TSL Tier-based Strictly Local

LTT Locally Threshold Testable

LT Locally Testable

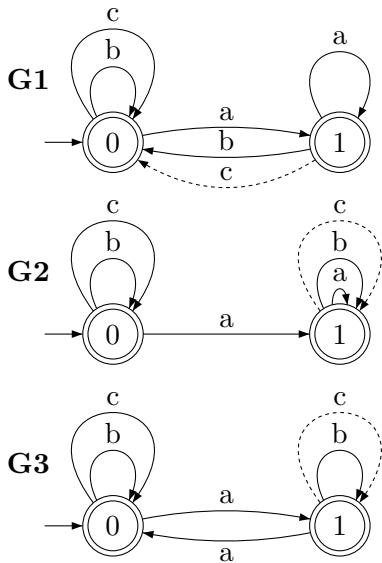
SL Strictly Local

PT Piecewise Testable

SP Strictly Piecewise

(McNaughton and Papert 1971, Simons 1975, Rogers et al. 2010, Heinz et al. 2011)

### 3 Examples



1. G1 generates/recognizes all words except those with a forbidden string [ac]
  2. G2 generates/recognizes all words except those with a forbidden subsequence [a...c]
  3. G3 generates/recognizes all words except those with a [c] whose left context has an even number of [a]s
- G1 is Strictly 2-Local, G2 is Strictly 2-Piecewise, and G3 is Counting.

## Harmony

Samala Chumash regressive sibilant harmony (Applegate 1972)

Alveolar		
/s-api-tʰo-us/	[sapitʰolus]	‘he has a stroke of good luck’
/s-ij-tifi-jep-us/	[sistisijepus]	‘they (2) show him’
Post-alveolar		
/s-api-tʰo-us-waf/	[ʃapitʰolufwaf]	‘he had a stroke of good luck’
/ha-s-xintila-waf/	[hafxintilawaf]	‘his former Indian name’

## Consonantal harmony patterns are Strictly 2-Piecewise

Strictly 2-Piecewise are those which describe patterns in terms of *permissible* and *forbidden subsequences* of length 2.

### Example

Phonotactic pattern derived from Samala Chumash

[ sapit<sup>h</sup>olus    sistisijepus    ʃapitʃ<sup>h</sup>olufwaf    hafxintilawaf ]

Notation:

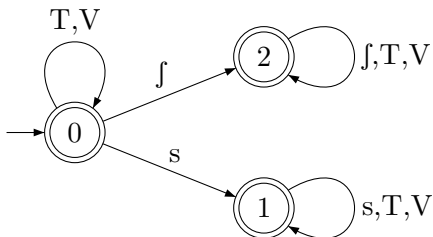
s	[+strident,+anterior]	T	[-syllabic,-strident]
ʃ	[+strident,-anterior]	V	[+syllabic]

**Forbidden** sʃ, ʃs

**Permissible** sT, sV, ʃT, ʃV, Ts, Tʃ, TV, TT, Vs, Vʃ, VT, VV  
(everything else)

(Heinz 2010)

## FSA representation of this infinite set



**Figure:** An automaton which recognizes the sibilant harmony pattern in Samala Chumash.

s	[+strident,+anterior]	T	[-syllabic,-strident]
ʃ	[+strident,-anterior]	V	[+syllabic]

(Rogers et al. 2010)

## Are unbounded stress patterns Strictly 2-Piecewise?

1. We can answer this by identifying the *permissible* and *forbidden* subsequences in LHOR.

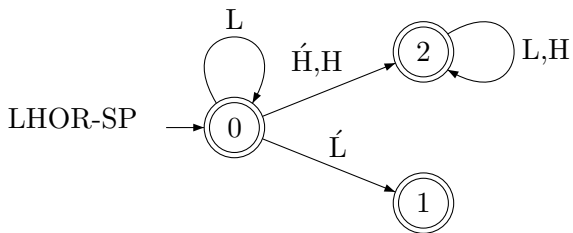
# The permissible and forbidden subsequences of LHOR

H	L	HL	HH	LH
LH	HLH	HLH	HLH	HLH
LHL	LHH	LLH	LLH	LHLH
LHHL	HLHL	HLHL	HLHL	HLHL
LHLL	LHLL	LHLL	LHLL	LHLL
HLHL	HLHL	HLHL	HLHL	HLHL
HLHL	HLHL	HLHL	HLHL	HLHL

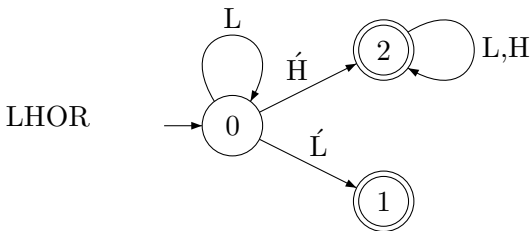
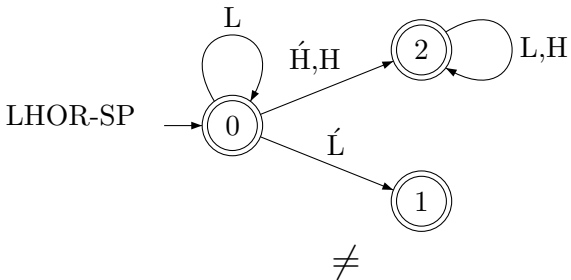
permissible		forbidden	
LL	HH	HH	LL
LH	HL	HL	HL
LH	HH	HH	LH
LH	HL	HL	LH



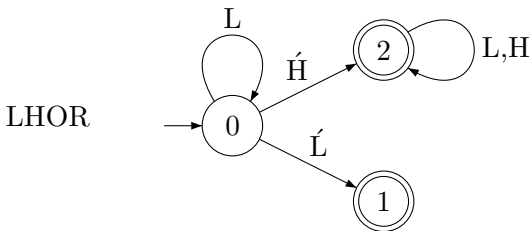
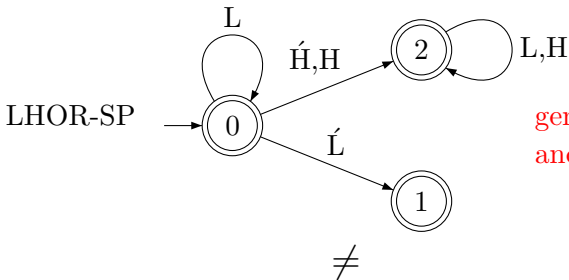
## FSA representations



## FSA representations



## FSA representations



## Interim Summary

- SP generalizations can express “At most one stress.” (e.g.  $\acute{H} \acute{H}$  is a forbidden subsequence)
- SP generalizations cannot express “At least one stress” or “exactly one stress.”

# Culminativity

- Culminativity is the principle that each word has exactly one prosodic peak.
- It has long been recognized as a central principle in virtually every theory of stress.

(Hyman 1977, Prince 1983, Halle and Verganud 1987, Idsardi 1992, Hayes95, Hulst et al. 2010)

## Culminativity as an infinite set

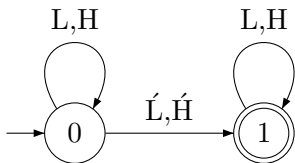


Figure: A finite-state acceptor describing Culminativity.

## All ingredients but one

1. The infinite set of LHOR
2. The infinite set of LHOR-SP
3. The infinite set of Culminativity
4. ... need some way to combine Culminativity with LHOR-SP to yield LHOR

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1. The infinite set of LHOR
2. The infinite set of LHOR-SP
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4. ...need some way to combine Culminativity with LHOR-SP to yield LHOR

Set intersection



## Set intersection

1. Set intersection yields a set including only those elements common to both.
2. Automata product computes set intersection for regular sets.

For automata  $A$ ,  $B$ ,  $C$ :

if

$$A \times B = C$$

then

$$L(A) \cap L(B) = L(C)$$

(Sipser 1997, Hopcroft et al. 2001)

## Specific claims in this talk

Language-theoretic version

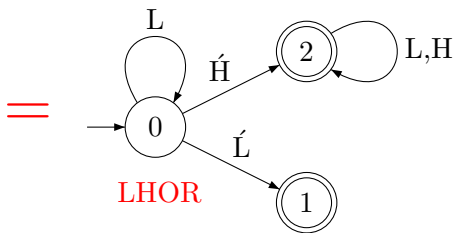
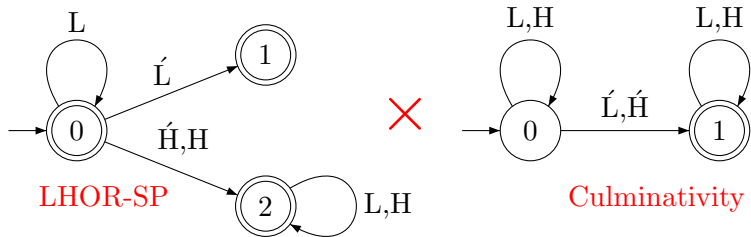
1.  $L(\text{LHOR-SP}) \cap L(\text{Culminativity}) = L(\text{LHOR})$
2.  $L(\text{LHOL-SP}) \cap L(\text{Culminativity}) = L(\text{LHOL})$
3.  $L(\text{RHOL-SP}) \cap L(\text{Culminativity}) = L(\text{RHOL})$
4.  $L(\text{RHOR-SP}) \cap L(\text{Culminativity}) = L(\text{RHOR})$

## Specific claims in this talk

Automata-theoretic version

1.  $A(\text{LHOR-SP}) \times A(\text{Culminativity}) = A(\text{LHOR})$
2.  $A(\text{LHOL-SP}) \times A(\text{Culminativity}) = A(\text{LHOL})$
3.  $A(\text{RHOL-SP}) \times A(\text{Culminativity}) = A(\text{RHOL})$
4.  $A(\text{RHOR-SP}) \times A(\text{Culminativity}) = A(\text{RHOR})$

All of these claims are easily verified.



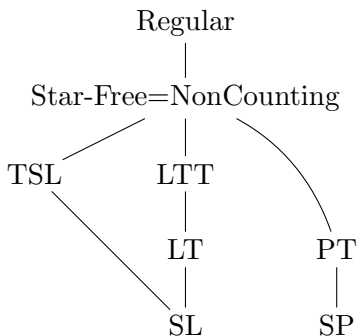
Harmony  $\times$  Culminativity = Unbounded Stress  
Patterns

$$\text{Harmony} = \frac{\text{Unbounded Stress Patterns}}{\text{Culminativity}}$$

## Conclusion

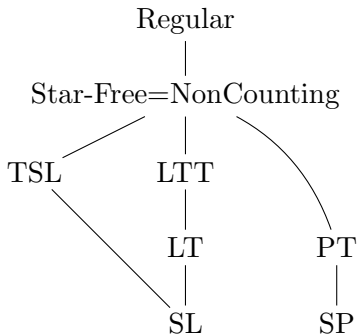
1. This analysis unifies long-distance phenomenon in unbounded harmony systems and simple segmental harmony systems: they are Strictly Piecewise modulo Culminativity.
2. It will be interesting to see how far this result can be pushed when more complicated unbounded stress patterns and segmental harmony patterns are considered.
3. There is also a learnability consequence since  $SP_2$  patterns can be learned from positive data (Heinz 2010a,b) and Culminativity has been argued to be a principle of UG.

## Conclusion



4. More generally, these computational classes allow one to identify the nature of phonological patterns, independent of any particular grammatical formalism.

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Thank you