Learning Left-to-Right and Right-to-Left Iterative Languages

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LRI and RLI Languages

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2. identifiable in the limit from positive data
3. essentially the classes obtainable by merging final and start states in prefix and suffix trees, respectively

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2. a step towards mapping out space of language classes obtainable by Muggleton's (1990) general state-merging IM1 algorithm
3. help reveal the algebraic structure underlying state-merging and the reverse operator
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- Rules or constraints governing **word well-formedness**
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Specific Sound Patterns

- No Triple Consonant Clusters in Yokuts:
  - Includes { ab, abba, ababa, . . . }
  - Excludes { bbb, abbb, abbba, bbbba, . . . }
- Symmetric Sibilant Harmony (Navajo):
  - Includes {sos, sototos, . . . ʃoʃ, ʃotototoʃ . . . }
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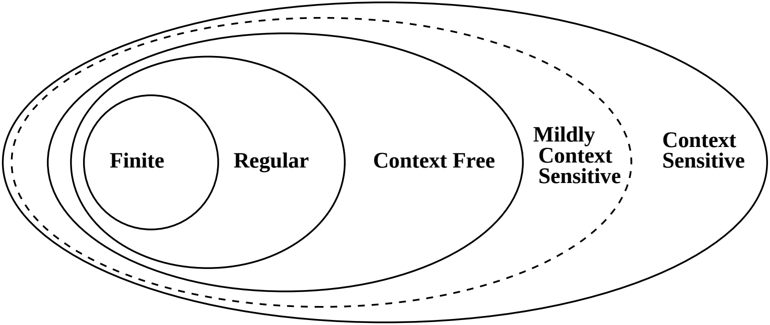

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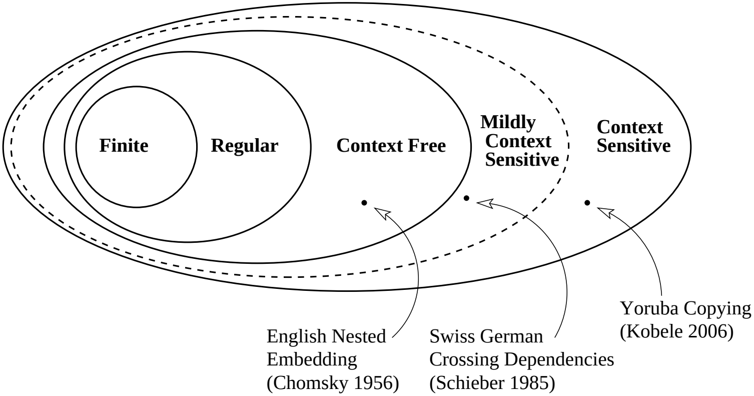

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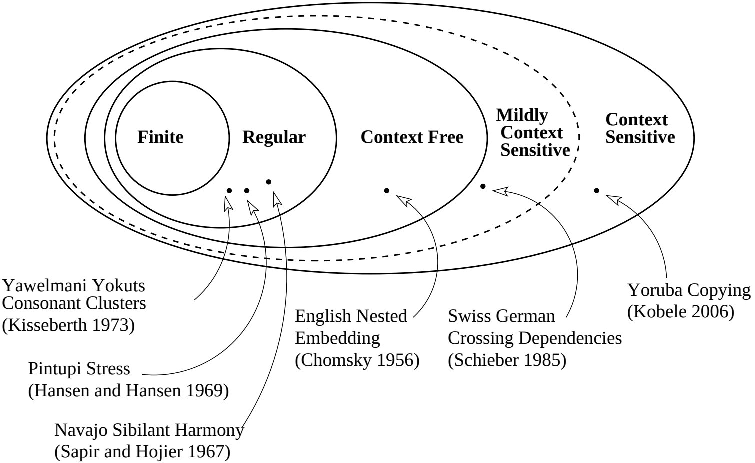
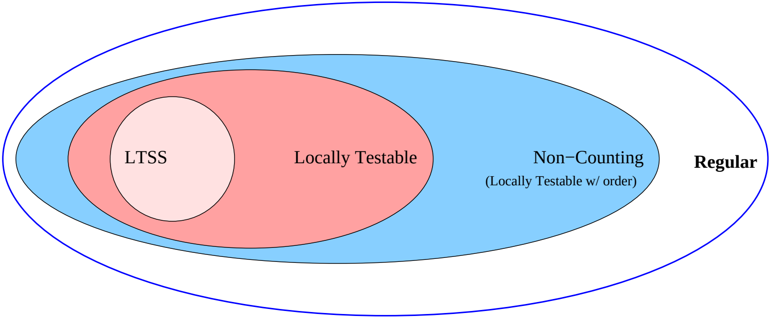


The Subregular Hierarchy



(McNaughton and Papert 1971, Pullum and Rogers 2007)

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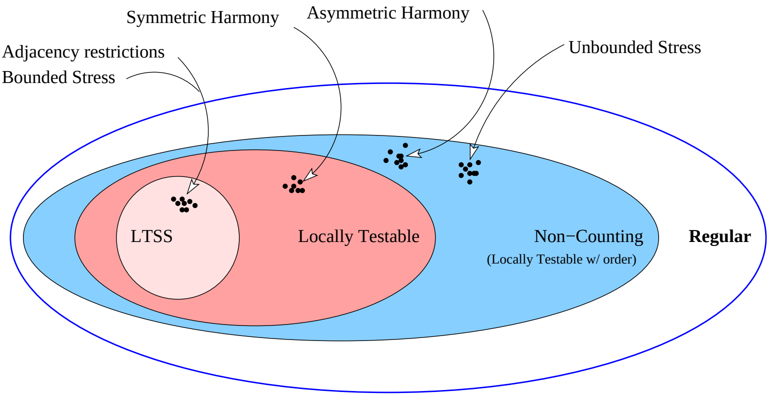

(Greenberg 1978, Hansson 2001, Hayes 1995)

Grammatical Inference of Regular Languages is Theoretical Phonology

- The properties of learnable subclasses of the regular languages are candidates as universal properties of sound patterns

  E.g. Angluin 1982, Muggleton 1990, Fernau 2003, . . .

- Which can be evaluated by comparing them to the
  1. Linguists' knowledge of the range of variation

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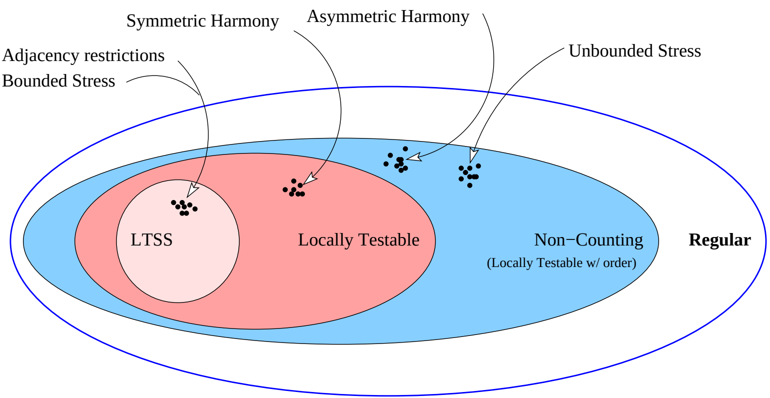

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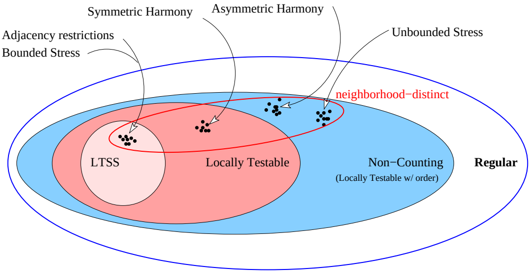

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  - all but 2 attested stress patterns $\subset$ 1-1 ND

(Heinz 2007)

LRI: Language-theoretic Characterization

- LRI languages are defined as the intersection of two classes of languages.

1. $\{L : \text{whenever } u, v \in L, T_L(u) = T_L(v)\}$
2. $\{L_1 \cdot L_2^* : L_1, L_2 \in \mathcal{L}_{fin}\}$

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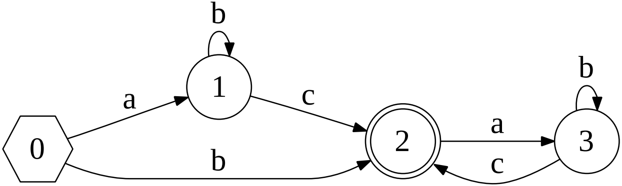

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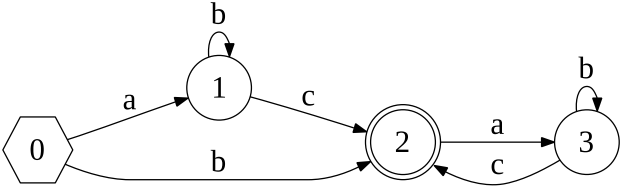

These languages I call 1-final deterministic

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A language L is **left-to-right iterative** iff

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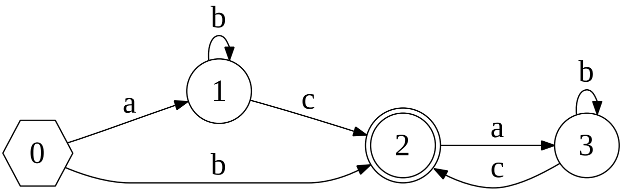

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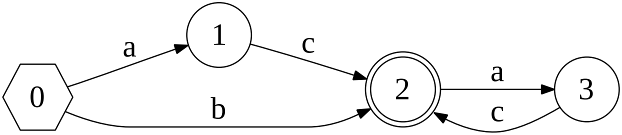

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$\Rightarrow$ Generally, given $u, uv \in L$, we infer $uv^* \subseteq L$.

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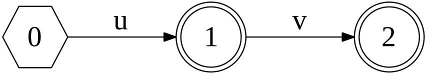

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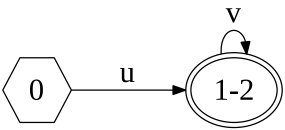

Learning LRI

$$\boxed{\phi(S) = PT(S)/\pi_{final}}$$

1. Merge final states in the prefix tree
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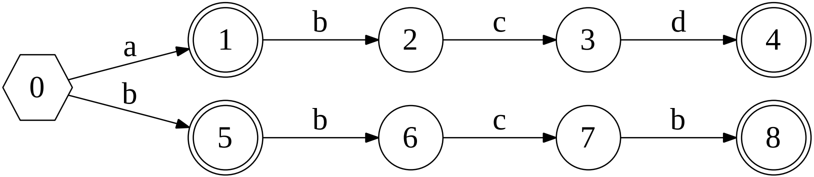
Illustration of Learning LRI

Sample = { a, b, abcd, bbcb}



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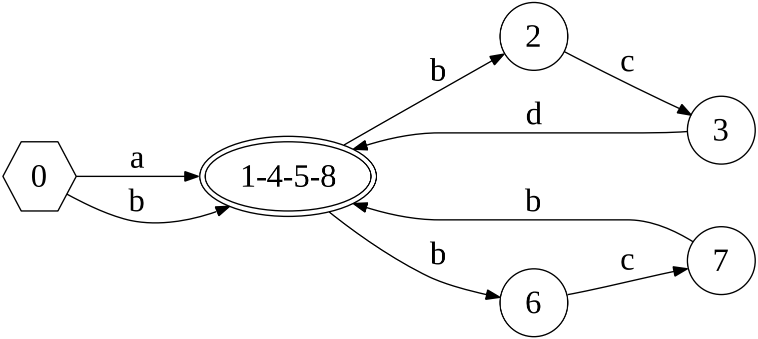

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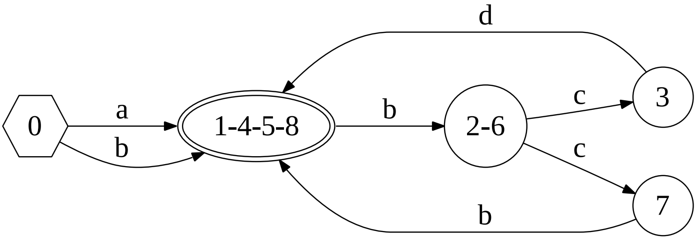

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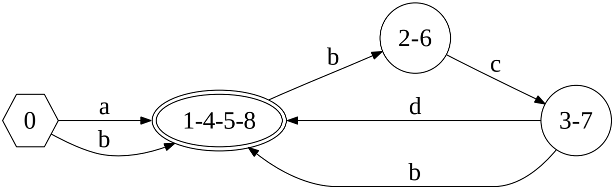

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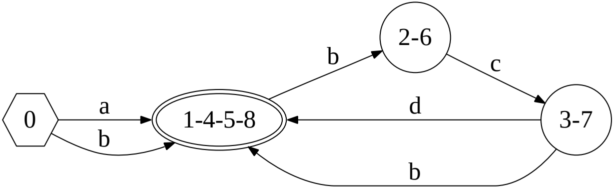

- The algorithm differs only from ZR (Angluin 1982) in that states are **not** merged to remove reverse non-determinism!

Learning Results for LRI

Theorem. Every $L \in LRI$ has a characterstic sample. Since $L = L_1 \cdot L_2^*$ where $L_1, L_2 \in \mathcal{L}_{fin}$, such a sample is

$$L_1 \cup L_1 \cdot L_2$$

Theorem. $L{=}L(PT(S)/\pi_{final})$ is the smallest language in LRI which includes S.

Theorem. The learner ϕ identifies LRI in the limit from positive data.

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  i.e. it crosscuts the Subregular Hierarchy
- Unknown if it is function-distinguishable (Fernau 2003)

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State-merging: Algorithm IM1

- Begin with a structured representation *PT* of the sample
- Use an equivalence relation to determine which states to merge
- The equivalence relation is determined by a function f

$$p \sim q \text{ iff } f(p) = f(q)$$

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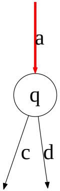

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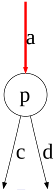

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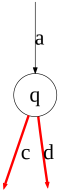

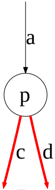

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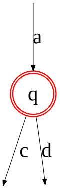

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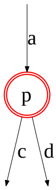

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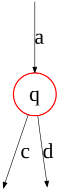

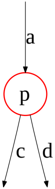

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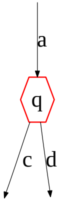

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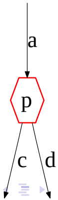

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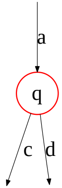

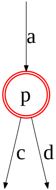

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Summary of known classes obtainable in this way

(Garcia et. al 1990)

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I_k	$(k+1)$ LTSS	?
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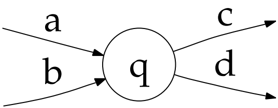
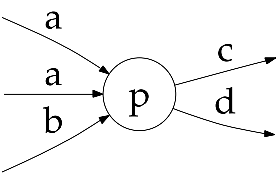
Neighborhood-distinctness





- The neighborhood of state is determined by the function:

$$nd_j^k(q) = (I_j(q), O_k(q), [\mathbf{q} \in \mathbf{F}], [\mathbf{q} \in \mathbf{I}])$$

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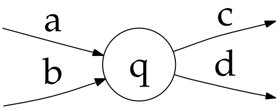

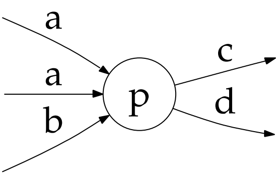

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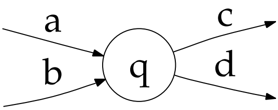

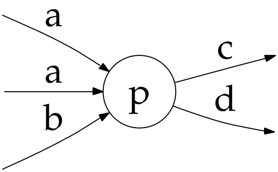

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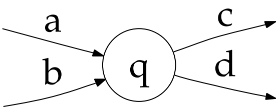

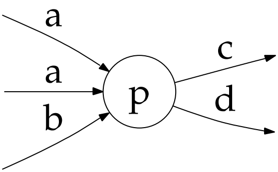

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  3. help reveal the algebra underlying state-merging algorithms and the reverse operator
- Phonotactic patterns are regular and it is an open question which of their properties are sufficient or necessary for learning
- The neighborhood-distinct hypothesis is one proposal
- The LRI and RLI languages are a small but necessary step towards a better understanding of this proposal

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RLI: Language-theoretic Characterization

- LRI languages are defined as the intersection of two classes of languages.

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$$\phi(S_t) = ST(S_t)/\pi_{start}$$

- Merge states to remove reverse non-determinism

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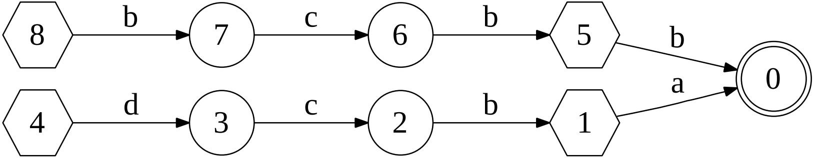
Illustration of Learning RLI

Sample = { a, b, dcba, bcbb}



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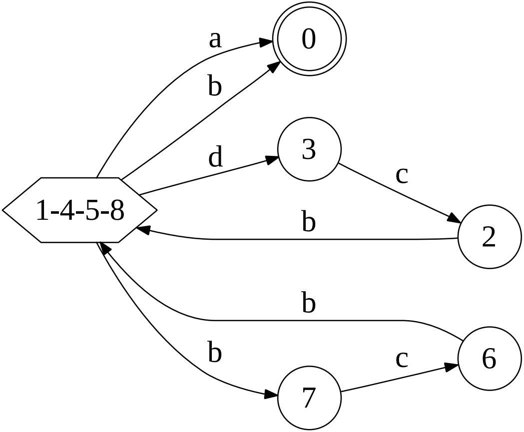

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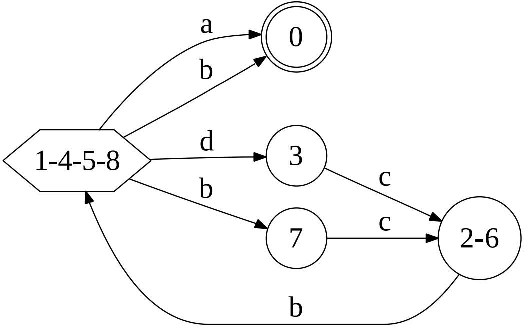

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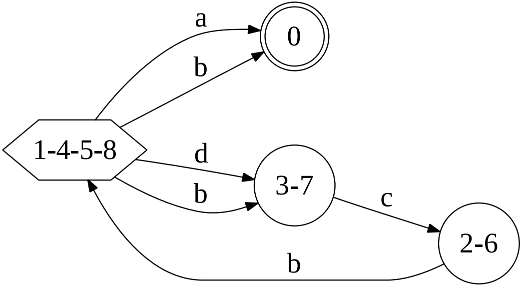

Asymmetric Harmony

- Sarcee is like Navajo except the pattern is asymmetric: [ʃ] may precede [s] in a word, but [s] cannot precede [ʃ]
  - Includes { sotos, ʃotoʃ, ʃotos, . . . }
  - Excludes { sotoʃ, . . . }
- This pattern is Noncounting.

(Hansson 2001)

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Bounded Stress Patterns

- Secondary stress falls on nonfinal odd syllables (counting from left)
- Primary stress falls on the initial syllable

a.	$\acute{\sigma}\ \sigma$	páɳa	'earth'
b.	$\acute{\sigma}\ \sigma\ \sigma$	tʲúʈaya	'many'
c.	$\acute{\sigma}\ \sigma\ \grave{\sigma}\ \sigma$	máɭawàna	'through from behind'
d.	$\acute{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \sigma$	púɭiŋkàlatʲu	'we (sat) on the hill'
e.	$\acute{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \grave{\sigma}\ \sigma$	tʲámulìmpatʲùŋku	'our relation'
f.	$\acute{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \sigma$	ʈíɭirìŋulàmpatʲu	'the fire for our benefit flared up'
g.	$\acute{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \grave{\sigma}\ \sigma$	kúranʲùlulìmpatʲùɻa	'the first one who is our relation'
h.	$\acute{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \grave{\sigma}\ \sigma\ \sigma$	yúmaɻìŋkamàratʲùɻaka	'because of mother-in-law'

Hayes (1995:62) citing Hansen and Hansen (1969:163)

Bounded Stress Patterns

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a.	**σ́** σ	páɳa	‘earth’
b.	**σ́** σ σ	t^{j}úʈaya	‘many’
c.	**σ́** σ **σ̀** σ	máɭawàna	‘through from behind’
d.	**σ́** σ **σ̀** σ σ	púɭiŋkàlat^{j}u	‘we (sat) on the hill’
e.	**σ́** σ **σ̀** σ **σ̀** σ	t^{j}ámulìmpat^{j}ùŋku	‘our relation’
f.	**σ́** σ **σ̀** σ **σ̀** σ σ	ʈíɭirìŋulàmpat^{j}u	‘the fire for our benefit flared up’
g.	**σ́** σ **σ̀** σ **σ̀** σ **σ̀** σ	kúran^{j}ùlulìmpat^{j}ùɻa	‘the first one who is our relation’
h.	**σ́** σ **σ̀** σ **σ̀** σ **σ̀** σ σ	yúmaɻìŋkamàrat^{j}ùɻaka	‘because of mother-in-law’

Hayes (1995:62) citing Hansen and Hansen (1969:163)

Unbounded Stress Patterns

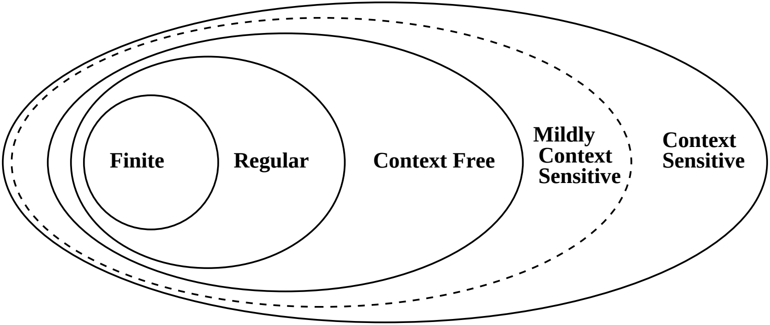
KwaKwala: Leftmost Heavy Otherwise Rightmost

- Stress the heavy syllable closest to the left edge. If there is no heavy syllable, stress the rightmost syllable.

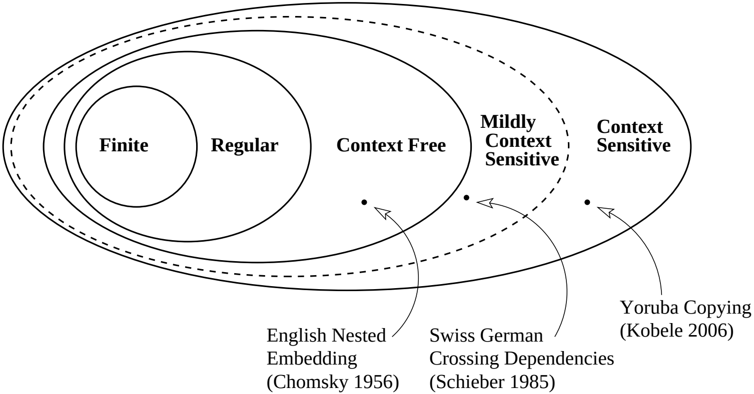
a.	**Ḣ́** H H	d.	L **Ĺ**
b.	L L **H́** L L	e.	L L **Ĺ**
c.	L L L **H́**	f.	L L L **Ĺ**

Walker (2000:21) citing Zec (1994)

Language Patterns and the Chomsky Hierarchy



Language Patterns and the Chomsky Hierarchy



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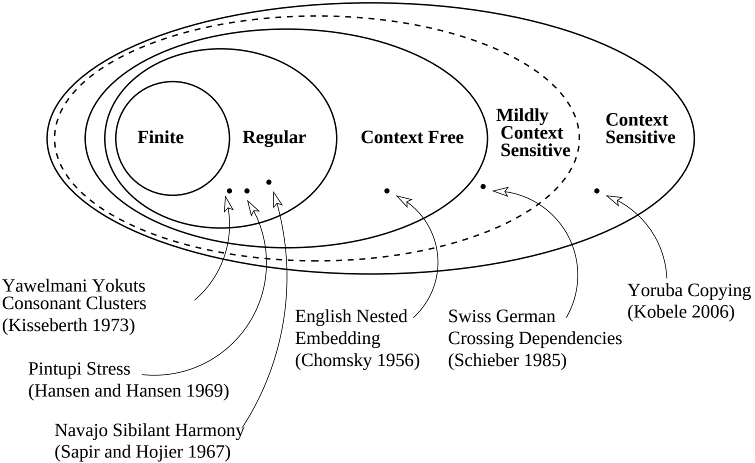

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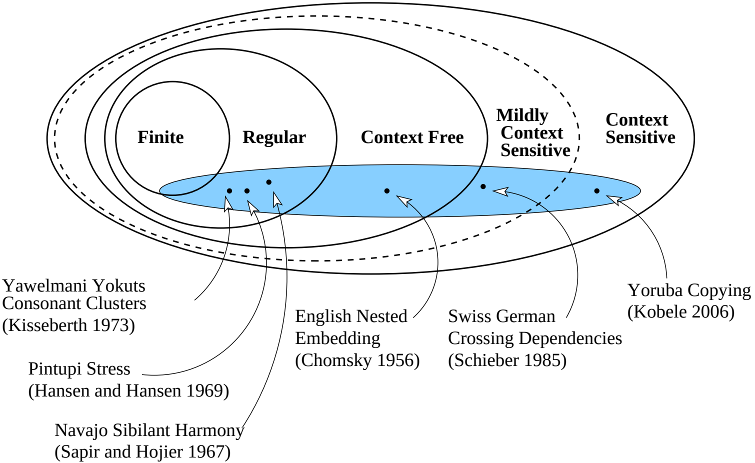

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