Weak Determinism and Simultaneous Application via Boolean Monadic Recursive Schemes

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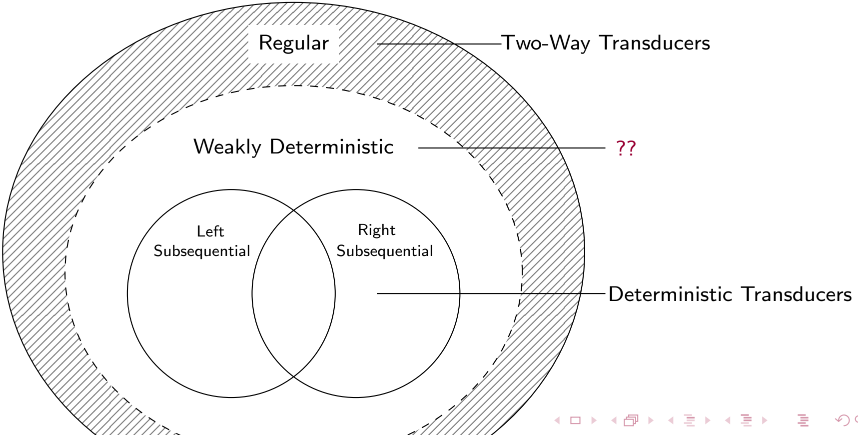
Phonology and Complexity

Phonological patterns are regular [Joh72, KK94].
But, we do not need the full expressivity of the Regular class to describe phonology. Most patterns are subregular.



Structure of Talk

- Weak Determinism; Empirical Motivations
- Logical Characterization; ‘Simultaneous Application’
- Sour Grapes is not Weakly Deterministic
- Conclusion

Subsequential

(1) Sibilant harmony in Ineseño Chumash

a. /s-api-tʃʰo-us-waʃ/
[ʃ-api-tʃʰo-uʃ-waʃ] 'He had a stroke of good luck'

b. /s-iʃ-tiʃi-jep-us/
[s-is-tisi-jep-us] 'They (two) show him'

(1) is *right subsequential*:

- [Deterministic] The *rightmost* sibilant determines the anteriority of all sibilants in a word.
- [Unbounded] The sibilant undergoing harmony can be separated from the rightmost one by any number of morphemes.

Beyond Subsequential

(2) Stem-Controlled ATR Harmony in Akan

a. *[-ATR] Root*

ɔ-tsɪrɛ-ɪ	'3S-show-3S.OBJ'
ɔ-bɛ-tʊ-ɪ	'3S-FUT-throw-3S.OBJ'

b. *[+ATR] Root*

o-fiti-i	'3S-pierce-3S.OBJ'
o-be-tu-i	'3S-FUT-dig-3S.OBJ

When determining the ATR value for the vowels:

- Vowels in suffixes depend on information to the left
- Vowels in prefixes depend on information to the right

Beyond Subsequential

(3) LHOL Stress in Lushootseed

a. LHLHL $\mapsto$ LH́LHL

b. HHHHH $\mapsto$ H́HHHH

c. LLLL $\mapsto$ ĹLLLL

Similar observations can be made about the stress pattern in (3).

When determining whether a syllable is stressed:

- Heavy syllables depend on information to the left
- (Initial) light syllables depend on information to the right

Phonology and Complexity

(2) and (3) are notably different from the Sour Grapes pathology:

(4) Sour Grapes

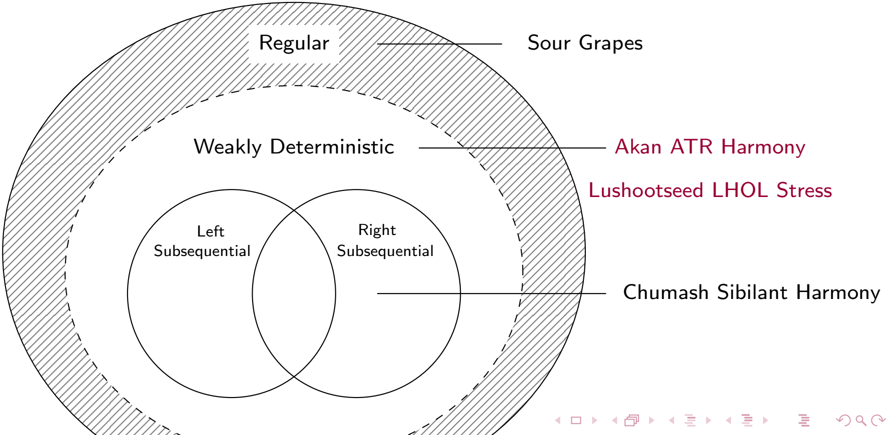
a.	$+ - - - - -$	$\mapsto$	$+ + + + + +$
b.	$+ - - - - \boxminus$	$\mapsto$	$+ - - - - \boxminus$
c.	$- - - - - -$	$\mapsto$	$- - - - - -$

(4) is unbounded circumambient [Jar16]:

- [Circumambient] Every input segment needs information about a trigger to the left and a blocker to the right.
- [Unbounded] There is no finite bound on how far the trigger and blocker can be from the segment undoing change.

Phonology and Complexity

Stem-controlled harmony and LHOL provide linguistic motivation for class of functions between subsequential and fully-regular, referred to as 'weakly deterministic' [Hei18, KJ20].



Why do we care?

1. 'Weakly deterministic' functions are linguistically meaningful.
   - The Weak Determinism Hypothesis [Hei18]
2. We do not have an agreed-upon formal definition of what constitutes a 'weakly deterministic' function.
   - Previous proposals: [HL13, MBMM18, MMBM21]
3. We do not have a definition that allows us to reason about what isn't weakly deterministic.
   - Conjectured that SG is not be Weakly Deterministic [HL13]

This talk addresses the points in (2) and (3) by providing such a definition and demonstration that Sour Grapes is outside the Weakly Deterministic boundary.

Towards a Logical Characterization

All regular functions can be expressed as the composition of a left and right subsequential function [EM65].

What distinguishes the weakly deterministic compositions?

We have two related notions of what makes a composition WD:

- Non-Interacting: [MBMM18]
- Disjunctive: The output value of every segment depends either on a left or a right subsequential function.

Both of these concepts are captured by the 'simultaneous application' operator introduced in this talk.

Simultaneous Application

$$\begin{cases} f: & a \to b/b\_\_ \\ g: & a \to b/\_\_b \end{cases}$$

x	a	b	a
	$g\downarrow$		
$g(x)$	b	b	a
			$f\downarrow$
$(f\circ g)(x)$	b	b	b

Table: Composition of f and g

x	a	b	a
	$g\downarrow$		$\downarrow f$
$(f\bullet g)(x)$	b	b. . .	b

Table: Simultaneous application of f and g

Simultaneous Application

Simultaneous application is the disjunctive application of two non-interacting functions:

- For every x in the input, if either f or g changes the value of x, the simultaneous $f \bullet g$ is the result of applying that change.

The idea: A function is weakly deterministic iff it can be expressed as the simultaneous application of a LS and a RS function.

This idea, when implemented in BMRS, gives us a logical characterization of weak determinism that

1. Matches the formal definition to the informal intention, and
2. Distinguishes weakly deterministic from non-deterministic ones

Boolean Monadic Recursive Schemes (BMRS)

BMRS is an abstract programming language that is used to model phonological patterns. See [CJ21].

Three key aspects of BMRS are relevant for this talk:

1. [Monadic Predicates] Input and output predicates over some alphabet are used to represent the function being modeled.
2. [If-then-else Expressions] The output for a string is determined by evaluating if-then-else statements.
3. [Predecessor and Successor] Functions over elements in the string that allow us to view information to the left and right.

A Quick Example

Word-final obstruent devoicing is modeled with the following BMRS expression (adapted from [CJ21]).

$$[voi]_o(x) = \text{if } \neg[son]_i(x) \wedge \ltimes_i(s(x)) \text{ then } \bot \text{ else } [voi]_i(x)$$

	⋊	b	æ	d	⋉
	1	2	3	4	5
$[son]_i(x)$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$[voi]_i(x)$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$[voi]_o(x)$	$\bot$	$\top$	$\top$	$\bot$	$\bot$
	⋊	b	æ	t	⋉

BMRS-Definition of Simultaneous Application

For every x in the input, if either P^f or P^g flips the truth value, then the simultaneous application $P^{f \bullet g}$ flips the truth value.

$P(x)$	$P^f(x)$	$P^g(x)$	$P^{f \bullet g}(x)$
$\top$	$\top$	$\top$	$\top$
$\top$	$\top$	$\bot$	$\bot$
$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\top$
$\bot$	$\top$	$\bot$	$\top$
$\bot$	$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$

$$P^{f \bullet g}(x) = \text{if } P(x) \text{ then } \left(P^f(x) \wedge P^g(x)\right) \text{ else } \left(P^f(x) \vee P^g(x)\right)$$

BMRS-Definition of Simultaneous Application

$$\begin{cases} f: & a \rightarrow b/b\_\_ \\ g: & a \rightarrow b/\_\_b \end{cases}$$

	a	b	a
$a(x)$	$\top$	$\bot$	$\top$
$b(x)$	$\bot$	$\top$	$\bot$
$a^f(x)$	$\top$	$\bot$	$\bot$
$b^f(x)$	$\bot$	$\top$	$\top$
$a^g(x)$	$\bot$	$\bot$	$\top$
$b^g(x)$	$\top$	$\top$	$\bot$
$a^{f \bullet g}(x)$	$\bot$	$\bot$	$\top$
$b^{f \bullet g}(x)$	$\top$	$\top$	$\bot$
	b	b	b

BMRS Definition of Weakly Deterministic

A function is weakly deterministic iff it can be modeled as the simultaneous application of two BMRS programs L and R where:

- L only uses predecessor
- R only uses successor

This definition is a BMRS implementation of the idea that WD functions can be decomposed into non-interacting left and right subsequential functions.

Case Study: LHOL Stress

(3) LHOL Stress in Lushootseed

a. LHLHL $\mapsto$ LH́LHL

b. HHHHH $\mapsto$ H́HHHH

c. LLLL $\mapsto$ ĹLLLL

For simplicity, we assume the following exist:

1. $\textsc{NoH-L}(x) = \top$ iff there are no Hs to the left of x
2. $\textsc{NoH-R}(x) = \top$ iff there are no Hs to the right of x

Decomposing LHOL

LHOL can be decomposed into the following two functions:

1. [Left Subsequential] Stress a heavy syllables if there are no other heavy syllables to the left of it.

$$\begin{aligned}\textsc{stress}^{L}(x) &= \text{if } H(x) \text{ then } \textsc{noH-L}(x) \text{ else } \bot \\ &\equiv H(x) \wedge \textsc{noH-L}(x)\end{aligned}$$

2. [Right Subsequential] Stress an intial light syllable if there are no heavy syllables to the right of it.

$$\textsc{Stress}^{R}(x) = L(x) \wedge \textsc{Initial}(x) \wedge \textsc{NoH-R}(x)$$

Simultaneous Application

The stress-assigning function for LHOL can be modeled by the simultaneous application of $\textsc{stress}^L$ and $\textsc{Stress}^R$.

In fact, the definition of simultaneous application simplifies to:

$$\textsc{stress}^L \bullet \textsc{Stress}^R = \textsc{stress}^L \vee \textsc{Stress}^R$$

Examples

	L	L	L	L
NoH-L(x)	$\top$	$\top$	$\top$	$\top$
NoH-R(x)	$\top$	$\top$	$\top$	$\top$
Stress^{L}(x)	$\bot$	$\bot$	$\bot$	$\bot$
Stress^{R}(x)	$\top$	$\bot$	$\bot$	$\bot$
Stress$^{L \bullet R}$(x)	$\top$	$\bot$	$\bot$	$\bot$
	Ĺ	L	L	L

	L	H	L	L	H
NoH-L(x)	$\top$	$\top$	$\bot$	$\bot$	$\bot$
NoH-R(x)	$\bot$	$\bot$	$\bot$	$\bot$	$\top$
Stress^{L}(x)	$\bot$	$\top$	$\bot$	$\bot$	$\bot$
Stress^{R}(x)	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
Stress$^{L \bullet R}$(x)	$\bot$	$\top$	$\bot$	$\bot$	$\bot$
	L	H́	L	L	H

Sour Grapes is not Weakly Deterministic

Consider a particular feature F that is spreading and the following three data points where $n, m \geq 0$.

$$\begin{array}{llcl} \text{(a)} & +(-)^n-(-)^m & \mapsto & +(+)^n+(+)^m \\ \text{(b)} & +(-)^n-(-)^m\boxminus & \mapsto & +(-)^n-(-)^m\boxminus \\ \text{(c)} & -(-)^n-(-)^m & \mapsto & -(-)^n-(-)^m \end{array}$$

Assume SG is weakly deterministic.
Then there must be two expressions F^L and F^R such that

- F^L only uses predecessor
- F^R only uses successor
- $F^{L \bullet R}$ models the input-output relations in (a)–(c).

Sour Grapes is not Weakly Deterministic

Let x_a, x_b, and x_c refer to the elements highlighted in red.

$$\begin{array}{llcl} \text{(a)} & +(-)^n-(-)^m & \mapsto & +(+)^n+(+)^m \\ \text{(b)} & +(-)^n-(-)^m\boxminus & \mapsto & +(-)^n-(-)^m\boxminus \\ \text{(c)} & -(-)^n-(-)^m & \mapsto & -(-)^n-(-)^m \end{array}$$

The data points in (a)–(c) are summarized by the following table.

	F	F^L	F^R	$\mathsf{F}^{L\bullet R}$
x_a	$\bot$			$\top$
x_b	$\bot$			$\bot$
x_c	$\bot$			$\bot$

We can reason backwards about F^L and F^R to get a contradiction.

Sour Grapes is not Weakly Deterministic

Because the truth value associated with F is flipped for x_a, either F^L or F^R must have flipped it.

$$\begin{array}{lll} \text{(a)} & +(-)^n-(-)^m & \mapsto & +(+)^n+(+)^m \\ \text{(b)} & +(-)^n-(-)^m\boxminus & \mapsto & +(-)^n-(-)^m\boxminus \end{array}$$

F^L only uses predecessor and therefore cannot distinguish between the environments in (a) and (b).

Thus, F^R must have flipped the truth value.

	F	F^L	F^R	F$^{L\bullet R}$
x_a	$\bot$		$\top$	$\top$
x_b	$\bot$			$\bot$
x_c	$\bot$			$\bot$

Sour Grapes is not Weakly Deterministic

F^R only uses successor and cannot distinguish between the environments in (a) and (c).

$$\begin{array}{llcl} \text{(a)} & +(-)^n-(-)^m & \mapsto & +(+)^n+(+)^m \\ \text{(c)} & -(-)^n-(-)^m & \mapsto & -(-)^n-(-)^m \end{array}$$

If it flips the truth value for x_a then it must also flip it for x_c.

	F	F^L	F^R	$F^{L\bullet R}$
x_a	$\bot$		$\top$	$\top$
x_b	$\bot$			$\bot$
x_c	$\bot$		$\top$	$\bot$

Now we have a contradiction.

The Big Picture

- Phonological patterns are subregular.
- We have empirical motivation a Weakly Deterministic class of functions that describes the range of phonological expressivity.
- Simultaneous application in BMRS gives us a logical characterization of this class.

Why BMRS?

- Definition of weakly deterministic functions without reference to composition.
- Principled way to reason about what functions are outside of the weakly deterministic boundary.
- Describe the logical structure of phonological patterns.

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