One (semi)ring to rule them all: Reconciling categorical and gradient models of phonotactics

LSA Session on Formal Language Theory in Morphology and Phonology

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What is this talk about?

Question: Are phonotactic grammars categorical or gradient?

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Answer: It depends on which semiring you use!

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1. Gradient phonotactic models account for new data from a Turkish acceptability judgment task better than categorical models.

2. This distinction turns out to be somewhat superficial if we think of models from a semiring-general perspective.

What is phonotactics?

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- /stik/ would be an ok English word; not a good Spanish word
- /ʧknɔw̃ɲʨ/ is a fine Polish word; not a good English word

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- “on a scale of 1-7, how likely is ‘steek’ to become an English word?”
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These judgments consistently display *gradience* [e.g. Chomsky and Halle, 1965, Coleman and Pierrehumbert, 1997, Scholes, 1966, Bailey and Hahn, 2001, Hayes and Wilson, 2008, Daland et al., 2011, a.o.].

What do we mean by gradience?

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poik

lvag

kip

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lvag $\ll$ poik $\ll$ kip

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However, the gradience observed in phonotactic acceptability judgments is largely predictable from "soft" versions of the same constraints that govern other phonological processes [Hayes, 2000].

Typical modeling approach is to use a grammar that produces a gradient output.

- Often based on statistical frequencies in the lexicon.

Implementing a gradient phonotactic grammar

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- **Proposal**: grammar is categorical and gradience comes from other sources.
- A categorical grammar labels words as either grammatical or ungrammatical

In particular, he claims that **categorical models do as well as or better than gradient models** in predicting phonotactic phenomena.

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- English medial cluster distributions [Gorman, 2013]

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2. Authors have different definitions of “categorical”
3. The gradient model used in (almost) all cases is the *UCLA Phonotactic Learner*

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For the sake of time I’m going to ignore these models.

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We'll adopt this definition of categorical.

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The UCLA Phonotactic Learner has become the poster boy for gradient phonotactics [Hayes and Wilson, 2008].

- But it also has to learn constraints from the data!
- Its performance is sensitive to how it is parameterized.
- Do categorical models outperform it because it is gradient? Because of its constraint selection process? Because it has been run with sub-optimal hyperparameters?

A simpler comparison

Let's compare the performance of two proposed categorical boolean models of Turkish vowel phonotactics against a simple probabilistic bigram model with a similar structure.

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We'll evaluate how these models predict new experimental data from a Turkish nonce word acceptability judgment task.

A new dataset of Turkish acceptability judgments

Turkish vowels

	[−back]		**[+back]**	
	[−round]	**[+round]**	**[−round]**	**[+round]**
[+high]	i	y	ɯ	u
[−high]	e	ø	a	o

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These constraints govern suffix allomorphy, but their effect is also detectable in the lexicon and in acceptability judgment tasks [Zimmer, 1969].

Our data

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- **Task**: Wug word acceptability judgments

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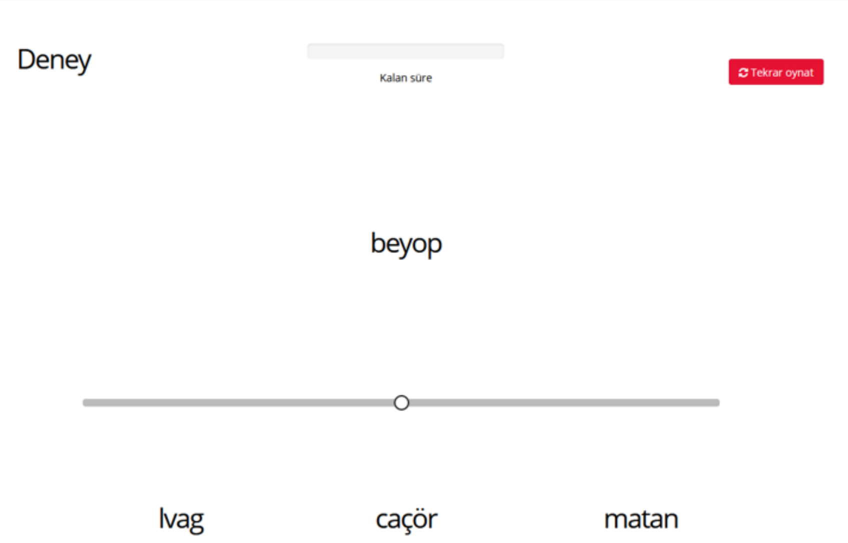
- Synthesized to speech using Google Cloud
- Words and recordings vetted by two native Turkish speakers

Experiment task



Analysis

Each participant rated 192 tokens after training and attention checks: 17,280 tokens.

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Responses are normalized to z-scores within participant

- Controls for differences in mean and spread between participants

Defining our models

We'll test three simple models that have similar structures:

Value type	**Constraint values**
Probability	Conditional probabilities
Boolean	Harmony [Gorman, 2013]
Boolean	Exception filtering [Dai, accepted]

General model structure

All the models are TSL-2 grammars that operate on the vowel tier

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- Informally, we **ignore consonants** and assign scores based on **vowel bigrams**

- Constraints can reference start and end symbols $\rtimes$ and $\ltimes$

Scoring bigrams

Each model type has a Δ function that assigns a value to a bigram.

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Probability model

$$\Delta_p : \Sigma^2 \rightarrow [0, 1]$$

Scoring words

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Boolean model: words are assigned 1 if they contain only legal bigrams, 0 otherwise

$$\text{bigram\_score}(x_1, \ldots, x_n) = \bigwedge_{i=1}^{n-1} \Delta_b(x_i, x_{i+1})$$

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Probability model: words are assigned the product of the probability of each bigram.

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$$\begin{aligned} \text{probability\_score}([\text{oi}]) &= \Delta_p(\rtimes\text{o}) \times \Delta_p(\text{oi}) \times \Delta_p(\text{i}\ltimes) \\ &= 0.08 \times 0.107 \times 0.458 \end{aligned}$$

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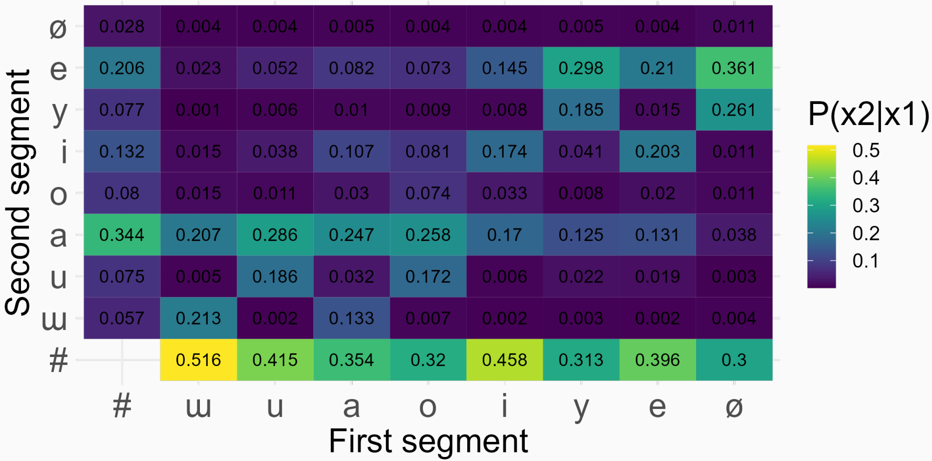
Choosing our values

How do we define Δ for each model?

Conditional probability model

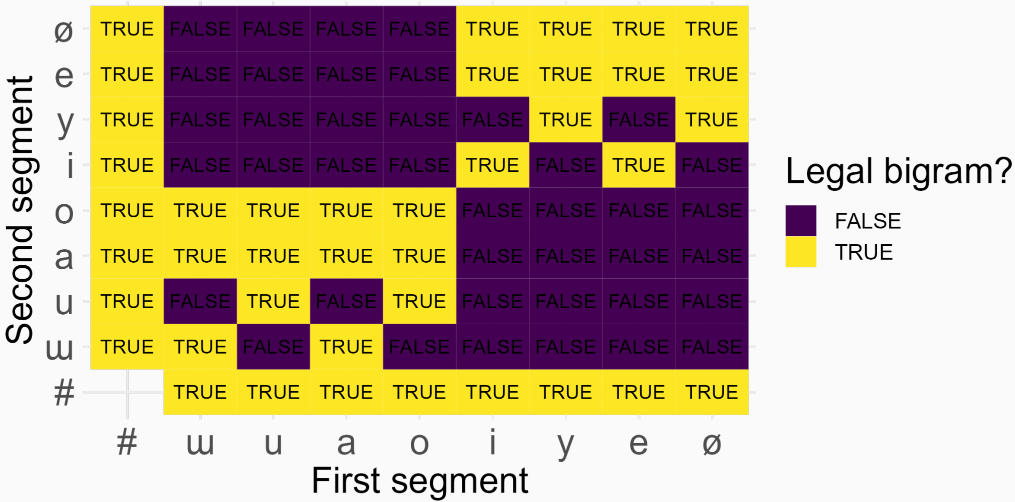
The probability model uses Laplace-smoothed conditional probabilities derived from 18,472 citation forms in the TELL database [Inkelas et al., 2000].



	#	ɯ	u	a	o	i	y	e	ø
ø	0.028	0.004	0.004	0.005	0.004	0.004	0.005	0.004	0.011
e	0.206	0.023	0.052	0.082	0.073	0.145	0.298	0.21	0.361
y	0.077	0.001	0.006	0.01	0.009	0.008	0.185	0.015	0.261
i	0.132	0.015	0.038	0.107	0.081	0.174	0.041	0.203	0.011
o	0.08	0.015	0.011	0.03	0.074	0.033	0.008	0.02	0.011
a	0.344	0.207	0.286	0.247	0.258	0.17	0.125	0.131	0.038
u	0.075	0.005	0.186	0.032	0.172	0.006	0.022	0.019	0.003
ɯ	0.057	0.213	0.002	0.133	0.007	0.002	0.003	0.002	0.004
#		0.516	0.415	0.354	0.32	0.458	0.313	0.396	0.3

Boolean harmony model [Gorman, 2013]

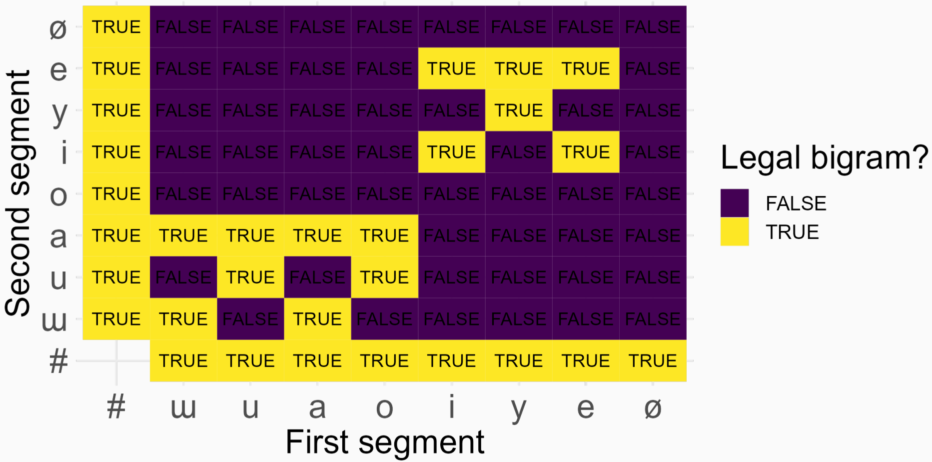
Words are grammatical if they satisfy both rounding and backness harmony.



	#	ɯ	u	a	o	i	y	e	ø
ø	TRUE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
e	TRUE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
y	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE
i	TRUE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE
o	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
a	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
u	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE
ɯ	TRUE	TRUE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE
#		TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

Boolean exception filtering model [Dai, accepted]

Categorical Turkish phonotactic grammar from Dai [accepted] learned via an exception filtering process.



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y	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE
i	TRUE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE
o	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
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Results

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Value type	**Constraint set**	r	τ	ρ
Probability	Conditional probabilities	**0.558**	**0.375**	**0.527**
Boolean	Harmony [Gorman, 2013]	0.371	0.303	0.369
Boolean	Exception filtering [Dai, accepted]	0.360	0.286	0.348

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The simple probabilistic model substantially outperforms the other models

Reconciling categorical and gradient models using semirings

The reconciliation begins

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But the basic structure of each model is the same:

- We assign some **value** to each segmental bigram
- We **aggregate** those values to get a score for the word

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$$\Delta\colon \Sigma^2 \longrightarrow \mathcal{R}$$

$$\text{score}(x_1 \ldots x_n) = \bigcirc\!\!\!\!\!\!\bigwedge_{i=1}^{n-1} \Delta(x_i, x_{i+1})$$

where $\mathcal{R}$ is some set of values and $\bigcirc\!\!\!\!\!\wedge$ is some binary operator over $\mathcal{R}$.

Other values of $\mathcal{R}$ and $\bigcirc\!\!\!\!\wedge$

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What does it compute?	$\mathcal{R}$	$\bigcirc\!\!\!\!\wedge$
Boolean scores [Gorman, 2013, Kostyszyn and Heinz, 2022, Dai, accepted]	$\{0, 1\}$	$\wedge$
Probabilities	$[0, 1]$	$\times$

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Probabilities	$[0,1]$	$\times$
Integer scores [Durvasula, 2020, Kostyszyn and Heinz, 2022]	$\mathbb{N}$	$+$

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Probabilities	$[0, 1]$	$\times$
Integer scores [Durvasula, 2020, Kostyszyn and Heinz, 2022]	$\mathbb{N}$	$+$
Constraint violation profiles	$\mathbb{N}^k$	$+$

Other values of $\mathcal{R}$ and $\bigcirc\!\!\!\!\wedge$

We can make these simple models compute even more interesting quantities!

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Boolean scores [Gorman, 2013, Kostyszyn and Heinz, 2022, Dai, accepted]	$\{0,1\}$	$\wedge$
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Left SL-2 string transduction	Σ^*	$+$

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This definition of our TSL-2 models is in **semiring-general terms**

$$\Delta\colon\ \Sigma^2 \longrightarrow \mathcal{R}$$

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We can parameterize our model with different semirings that provide implementations of $\mathcal{R}$ and $\bigotimes$.

What's a semiring?

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Monoid: a set $\mathcal{R}$ closed under a binary relation $\bigcirc\!\!\!\!\wedge$ such that:

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- There's an identity element $\top$ in $\mathcal{R}$ such that $a \bigcirc\!\!\!\!\wedge \top = \top \bigcirc\!\!\!\!\wedge a = a$

Semirings

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such that:

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- $x \bigowedge \bot = \bot \bigowedge x = \bot$

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We can separate the structure of the model from the values it computes.

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- Giorgolo and Asudeh [2014] apply different semirings to the same underlying semantic model to capture differences in heuristic vs. mathematical reasoning.

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Connecting the grammar to different domains

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- Harmony is a gradient preference when determining word acceptability
- **But both sensitive to the same configurations!**

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- operate on the vowel tier

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The same applies to other representations or grammars.

Closing remarks

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- "focus on what's a possible constraint or rule"; and
- "commit to a specific set of representations"

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- Constraints and representations in the grammar can be studied independently of the values the grammar assigns.
- Insight into the structure of the grammar can come from both gradient and categorical analyses!
- This flexibility allows our models to engage with a broader range of empirical phenomena.

Thank you!

Thanks to Huteng Dai, Jon Rawski, Megha Sundara, and Richard Futrell for many interesting discussions, and to my Turkish consultants Cem Babalik and Defne Bilhan.

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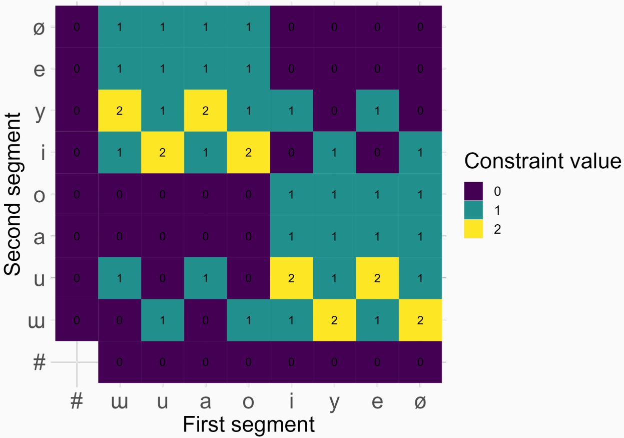
Stimuli structure

Cost semiring



	#	ɯ	u	a	o	i	y	e	ø
ø	0	1	1	1	1	0	0	0	0
e	0	1	1	1	1	0	0	0	0
y	0	2	1	2	1	1	0	1	0
i	0	1	2	1	2	0	1	0	1
o	0	0	0	0	0	1	1	1	1
a	0	0	0	0	0	1	1	1	1
u	0	1	0	1	0	2	1	2	1
ɯ	0	0	1	0	1	1	2	1	2
#		0	0	0	0	0	0	0	0

Results

Value type	**Constraint set**	r	τ	ρ
Probability	Conditional probabilities	**0.558**	**0.375**	**0.527**
Boolean	Cost [Durvasula, 2020, Kostyszyn and Heinz, 2022]	-0.379	-0.305	-0.386
Boolean	Harmony [Gorman, 2013]	0.371	0.303	0.369
Boolean	Exception filtering [Dai, accepted]	0.360	0.286	0.348