Input and Output Locality and Representation

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Locality & Autosegmental Representations

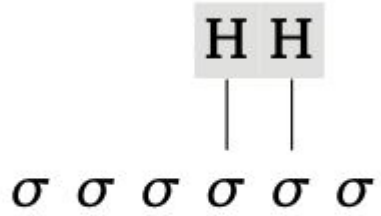
- Locality in **Theoretical Computational Linguistics:**
  - A computation is local if it operates over **some contiguous set of *k* positions in a string**
  - However, **long-distance phenomena widely attested** in phonology
- **Autosegmental Representations:**
  - Way to view **apparently long-distance processes as local** on some level (tier)
  - Power = “asynchronicity of **distinct tiers and manipulation of the association** relation between them”

Previous Work: Phonotactics

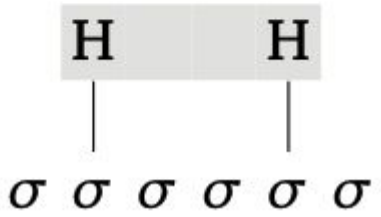
- Project tier to **capture long-distance patterns as local**
- Example: ***CLASH:** two high-tone segments adjacent on the tonal tier

a. *HH

b.



c.



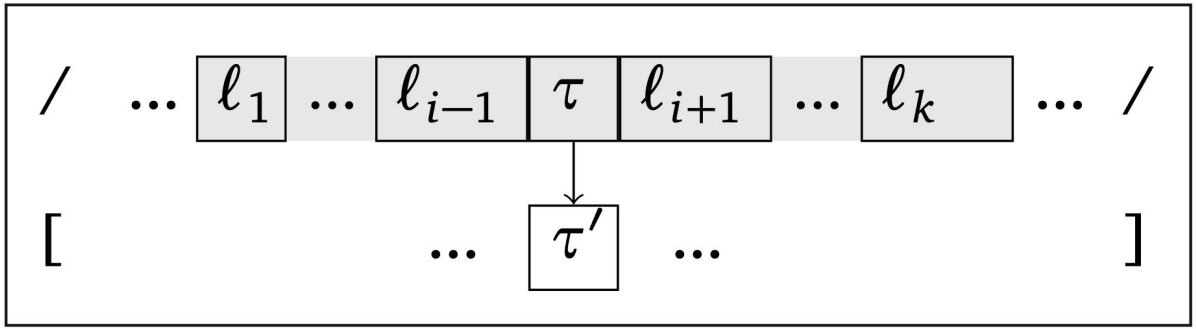
Proposal

- Extend phonotactic ideas to **phonological processes**
  - Model the **processes as functions**
  - **Test on tone** because there are a **range of interesting local & non-local processes**
- 4-way distinction of locality in tone processes
  - **ISL vs. RSL:** are changes conditioned on the input or the output?
  - **Autosegmental vs. String-based:** are we projecting tiers or considering just the string?

	Input	**Output**
Strings	ISL	RSL
ARs	A-ISL	A-RSL

Input Strictly Local (ISL) Functions

- Determine an output string for a given input string based only on **input substrings of length *k***
  - *k* = upper bound of **how much of the input string can be used to determine the output**
- Model logically
  - **Output predicates:** determine the content of positions in the output string from input
  - **Quantifier-Free Logic:** finite context accessed through *p(x)* and *s(x)*



ISL Functions: Rimi

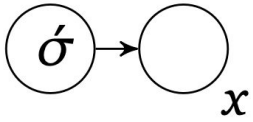
Tone shifts one TBU to the right (bounded shift)

a.	/u-hang-a/	[u-hang-a]	‘to meet’	$\sigma\sigma\sigma \mapsto \sigma\sigma\sigma$
b.	/u-pǫ́m-a/	[u-pǫm-á]	‘to go away’	$\sigma\acute{\sigma}\sigma \mapsto \sigma\sigma\acute{\sigma}$
c.	/mu-ntu/	[mu-ntu]	‘person’	$\sigma\sigma \mapsto \sigma\sigma$
d.	/rá-mu-ntu/	[ra-mú-ntu]	‘of a person’	$\acute{\sigma}\sigma\sigma \mapsto \sigma\acute{\sigma}\sigma$
e.	/u-huvi-ị/	[u-huvi-ị]	‘belief’	$\sigma\sigma\sigma \mapsto \sigma\sigma\sigma$
f.	/mu-tém-ị/	[mu-tem-í]	‘chief’	$\sigma\acute{\sigma}\sigma \mapsto \sigma\sigma\acute{\sigma}$

ISL Functions: Rimi

Bounded shift (ISL) (part 1)

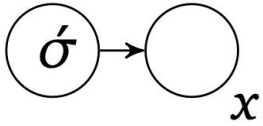
a. $\acute{\sigma}_o(x) \overset{\text{def}}{=} \acute{\sigma}(p(x))$

b. 

An output element *x* should bear a high tone if its predecessor *p*(*x*) in the *input* bears a high tone.

Bounded shift (ISL) (part 2)

a. $\sigma_o(x) \overset{\text{def}}{=} \neg\acute{\sigma}(p(x))$

b. not 

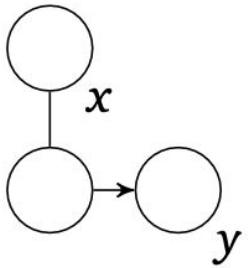
An output element *x* should be unspecified for tone if its predecessor *p*(*x*) in the *input* does not bear a high tone.

Autosegmental Input Strictly Local (A-ISL) Functions

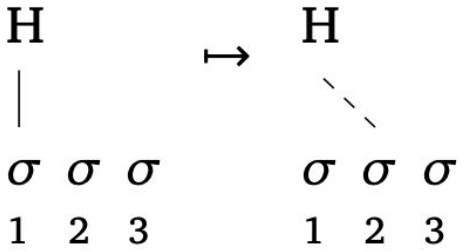
- **ARs:** two strings (one tone, one TBU), each with own *p* & *s* functions
  - **Association relation *A*** defines which tones are linked to which TBUs
- **Rimi bounded shift:** change in *A* between input & output

Bounded shift (A-ISL)

a. $A_o(x, y) \stackrel{\text{def}}{=} A(x, p(y))$

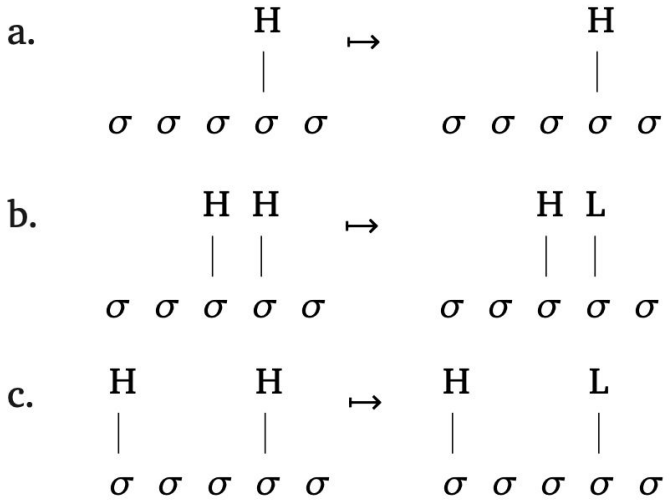
b. 

TBU *y* is associated with tone *x* in the output if *x* was associated with *y*'s predecessor in the input



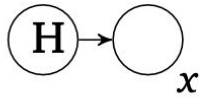
A-ISL Functions: Meeussen's Rule

- **Meeussen's Rule:** H ➔ L after another H, regardless of #intervening TBUs
  - **Not ISL:** unbounded search for previous H
  - **A-ISL:** two H's will be **adjacent on the tonal tier**



Long-distance Meeussen's rule (A-ISL)

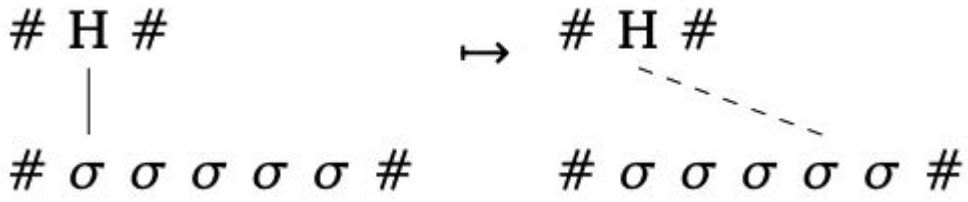
a. $L_o(x) \stackrel{def}{=} H(p(x))$

b. 

Zigula Unbounded Tone Shift: A-ISL

Underlying H tone shifts to penultimate TBU in the word regardless of how many TBUs intervene

a.	/ku-songoloz-a/	[ku-songoloz-a]	‘to avoid’
b.	/á-songoloz-a/	[a-songolóz-a]	‘he/she is avoiding’
c.	/ku-lómbez-a/	[ku-lombéz-a]	‘to ask’
d.	/ku-lómbez-ez-an-a/	[ku-lombez-ez-án-a]	‘to ask for each other’



Zigula Unbounded Tone Shift: A-ISL

$$A_o(x,y) \stackrel{\text{def}}{=} \mathrm{H}(x) \wedge \#(s(x)) \wedge \sigma(y) \wedge \#(s(s(y)))$$

Associate output positions *x* and *y* iff

x is a high tone

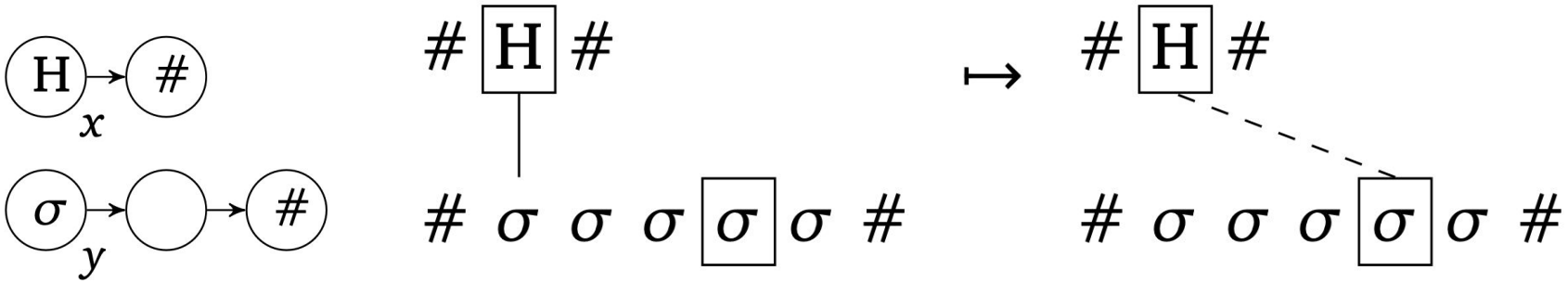
x is one position away from the end of the tonal tier

y is a TBU

y is two positions away from the end of the TBU tier

Zigula Unbounded Tone Shift: A-ISL

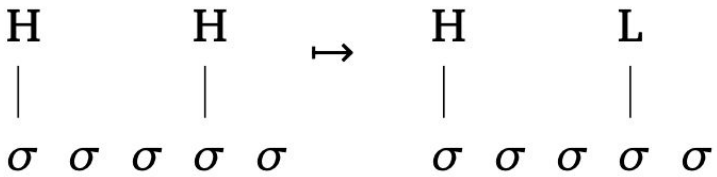
$$A_o(x,y) \stackrel{\text{def}}{=} \mathrm{H}(x) \wedge \#(s(x)) \wedge \sigma(y) \wedge \#(s(s(y)))$$



ISL vs. A-ISL Functions

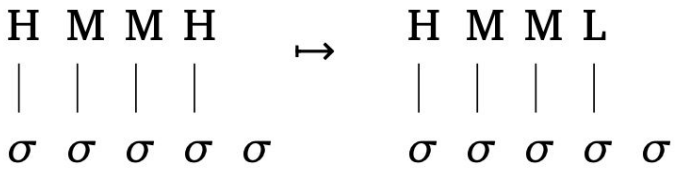
Theorem: If AR map is A-ISL, then the individual map on each tier is ISL

A-ISL version of long-distance Meeussen's rule



Tone tier is ISL

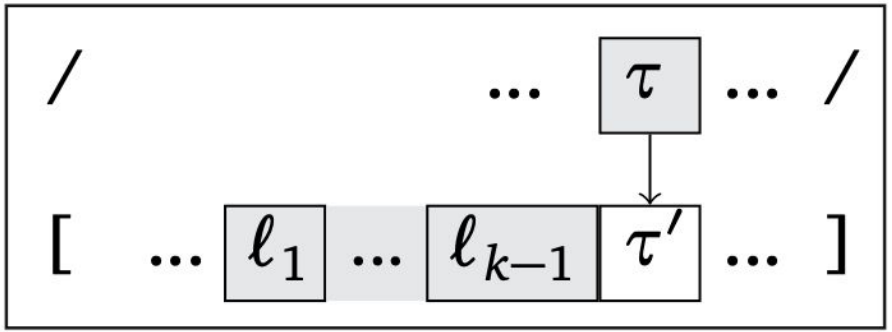
Non-A-ISL version of long-distance Meeussen's rule



Tone tier not ISL

Recursive Strictly Local (RSL) Functions

- Based on **Output Strictly Local** functions
- Determine an output string for a given input string based only on **output substrings of length *k***
  - *k* = upper bound of **how much of the input string can be used to determine the output**



RSL Functions: Recursion

- Use recursion in the **output association relation**
  - **Critical difference** between ISL & RSL
- Limit recursive logical definitions:
  - **Quantifier Free First Order logic**
  - **Directional:** transduction includes either formulas that use only *p* or those that use only *s*
  - **Only look at current input:** whenever a formula uses *p* or *s* it does so in a recursive predicate

RSL Functions: Unbounded Spreading

Spread high tone (H) to the end of a string

Base case: input tone is high

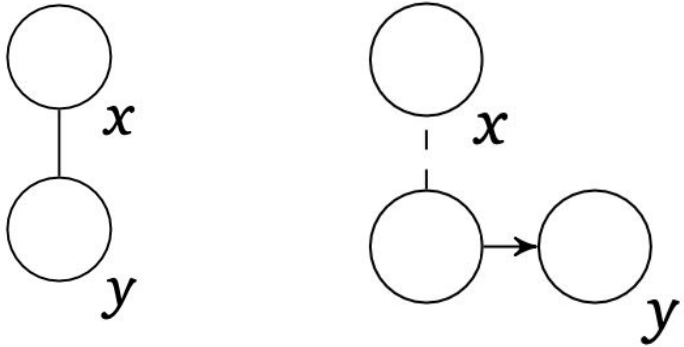
$$\acute{\sigma}_o(x) \overset{\text{def}}{=} \acute{\sigma}(x) \vee \acute{\sigma}_o(p(x))$$

Recursive step: previous output is high

Autosegmental Recursive Strictly Local (A-RSL) Functions

Unbounded spreading

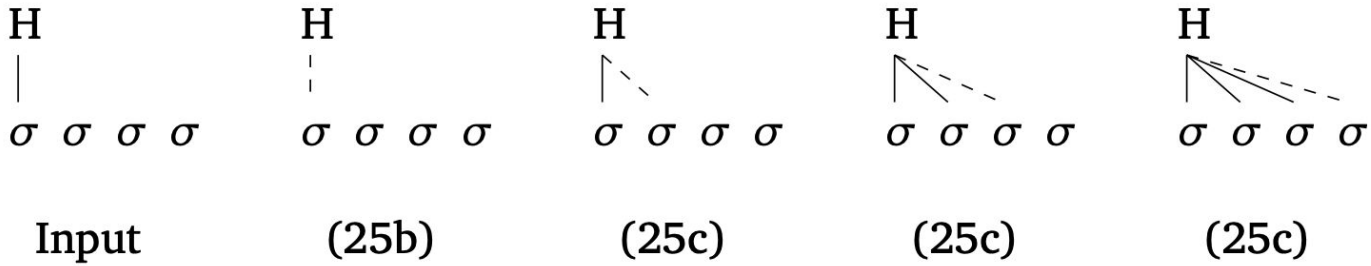
a. $A_o(x, y) \stackrel{\text{def}}{=} \underbrace{A(x, y)}_{\text{Base case}} \vee \underbrace{\left(A_o(x, p(y))\right)}_{\text{Inductive case}}$



A-RSL Functions: Unbounded Spreading

Unbounded spreading

a. $A_o(x, y) \stackrel{\text{def}}{=} \underbrace{A(x, y)}_{\text{b.}} \vee \underbrace{\left(A_o(x, p(y))\right)}_{\text{c.}}$



Summary Autosegmental Locality

Type of QF FO logical transduction

Representation

	Non-Recursive	**Recursive**
Strings	ISL	RSL
ARs	A-ISL	A-RSL

Goal = use these classes to establish computational properties of a phonological process & address larger questions of how representations interact with locality

Survey of tone processes that represent a range of QF FO logical transductions

PROCESS	ISL	A-ISL	RSL	A-RSL
Bounded spread (Bemba)	✓	✓	×	×
Bounded shift (Kuki-Thaadow)	✓	✓	×	×
Unbounded shift to penult (Zigula)	×	✓	×	✓
Unbounded spread to penult (Shambaa)	×	×	×	×
Unbounded Meeussen's, deletion (Arusa)	×	✓	×	×
Bounded Meeussen's, lowering (Luganda)	✓	×	×	×
Alternating Meeussen's, lowering (Shona)	×	×	✓	✓

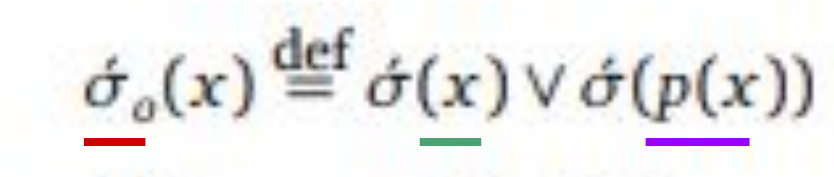
Bounded spread (Bemba; *Niger-Congo*)

Linguistic generalization: an H tone spreads exactly one TBU to the right

a.	/tu-la-kak-a/	[tu-la-kak-a]	‘we tie up’
b.	/bá-la-kak-a/	[bá-lá-kak-a]	‘they tie up’
c.	/bá-ka-fik-a/	[bá-ká-fik-a]	‘they will arrive’
d.	/bá-ka-bil-a/	[bá-ká-bil-a]	‘they will sew’

Bounded spread (Bemba; *Niger-Congo*): **ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right



The corresponding output, *x*, is H iff:

1. **The corresponding input *x* is H;**

   OR

2. **its predecessor is H.**

Bounded spread (Bemba; *Niger-Congo*): **ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right

Input	Output
/bá–……../	[bá–……..]
/bá–la-kak-a/	[bá–lá–……..]
/bá–la-kak-a/	[bá–lá–kak–..] (No change)
/bá–la-kak-a/	[bá–lá–kak–a] (No change)

The corresponding output *x* is H iff:

1. **The corresponding input *x* is H;**

   OR

2. **its predecessor is H.**

Bounded spread (Bemba; *Niger-Congo*): **ISL**

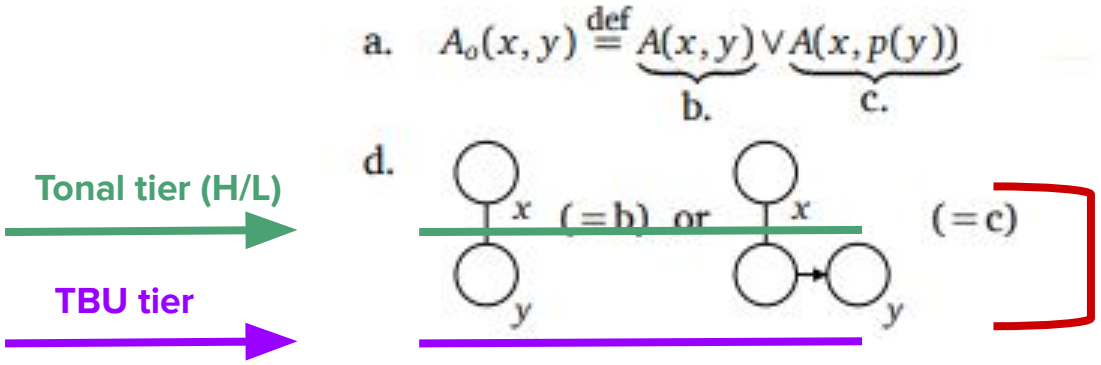
Linguistic generalization: an H tone spreads exactly one TBU to the right

- If this were RSL, we would get the wrong output and a type of unbounded spread:

Output
[bá–.....]
[bá–lá–.....]
*[bá–lá–kák–..]
*[bá–lá–kák–á]

Bounded spread (Bemba; *Niger-Congo*): **A-ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right



A(x,y):

"Associate the input x in the tonal tier with the input y in the TBU tier."

Bounded spread (Bemba; *Niger-Congo*): **A-ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right

---

a. $A_o(x,y) \overset{\text{def}}{=} \underbrace{A(x,y)}_{\text{b.}} \vee \underbrace{A(x,p(y))}_{\text{c.}}$

***x* and *y* are associated in the output iff:**

(b.) *x* and *y* are associated in the input

OR

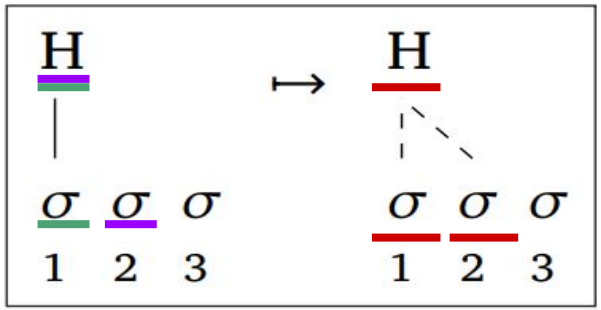
(c) *x* and the predecessor of *y* are associated in the input.

Bounded spread (Bemba; *Niger-Congo*): **A-ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right

The *x* tier is the tonal tier →

The *y* tier is the TBU tier →



***x* and *y* are associated in the output iff:**

(b.) *x* and *y* are associated in the input

OR

(c) *x* and the predecessor of *y* are associated in the input.

Bounded spread (Bemba; *Niger-Congo*): **A-ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right

Input	Output
#H……# bá–……#	#H……# bá–……
#H……# bá–la……#	#H……# bá–lá……
#H……# bá–la–kak–a#	#H……# bá–lá–kak–a#

***x* and *y* are associated in the output iff:**

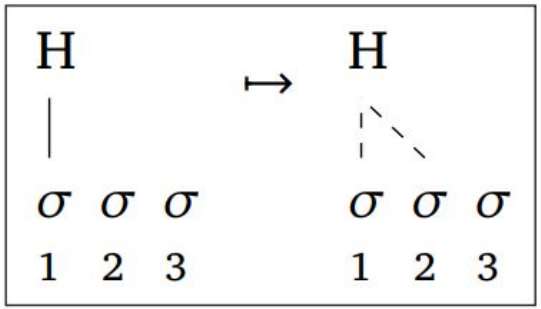
(b.) *x* and *y* are associated in the input

OR

(c) *x* and the predecessor of *y* are associated in the input.

Bounded spread (Bemba; *Niger-Congo*): **ISL & A-ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right



- Both formulas for ISL and A-ISL are QF and non-recursive.
- This analysis assumes that the underlying TBUs are either H or unspecified.
- Bounded spread is necessarily ISL, not RSL, given that it requires keeping track of how far the spreading has gone.

Survey of tone processes

PROCESS	ISL	A-ISL	RSL	A-RSL
Bounded spread (Bemba)	✓	✓	×	×
Bounded shift (Kuki-Thaadow)	✓	✓	×	×
Unbounded shift to penult (Zigula)	×	✓	×	✓
Unbounded spread to penult (Shambaa)	×	×	×	×
Unbounded Meeussen's, deletion (Arusa)	×	✓	×	×
Bounded Meeussen's, lowering (Luganda)	✓	×	×	×
Alternating Meeussen's, lowering (Shona)	×	×	✓	✓

Unbounded Meeussen's, deletion (Arusa; *Eastern Nilotic*): **A-ISL**

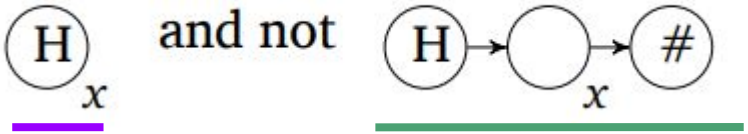
Linguistic generalization: the last H in a phrase is deleted, following another H, no matter the distance

/síday/	[síday]	'good'
/enkér síday/	[enkér siday]	'good chair'
/olórika síday/	[olórika siday]	'good ewe'

Unbounded Meeussen's, deletion (Arusa; *Eastern Nilotic*): **A-ISL**

Linguistic generalization: the last H in a phrase is deleted, following another H, no matter the distance

$$\mathrm{H}_o(x) \stackrel{\text{def}}{=} \mathrm{H}(x) \wedge \neg\big(\mathrm{H}(p(x)) \wedge \#(s(x))\big)$$



The output, *x*, is a H TBU iff:

1. **The input *x* is a H TBU**

AND

2. **Its predecessor is not a H TBU** AND **it is not followed by the right edge of the word.**

Unbounded Meeussen's, deletion (Arusa; *Eastern Nilotic*): **A-ISL**

Linguistic generalization: the last H in a phrase is deleted, following another H, no matter the distance

Input	Output
#en……..#	#en……# (No change)
#en–kér#……#	#en–kér#……# (No change)
#en–kér#sí…….#	#en–kér#sí…….# (i no longer H)
#en–kér#sí–dáy#	#en–kér#si–day# (a no longer H)

The output, *x*, is a H TBU iff:

1. **The input *x* is a H TBU**

AND

2. **Its predecessor is not a H TBU** AND **it is not followed by the right edge of the word.**

Survey of tone processes

PROCESS	ISL	A-ISL	RSL	A-RSL
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Bounded shift (Kuki-Thaadow)	✓	✓	×	×
Unbounded shift to penult (Zigula)	×	✓	×	✓
Unbounded spread to penult (Shambaa)	×	×	×	×
Unbounded Meeussen's, deletion (Arusa)	×	✓	×	×
Bounded Meeussen's, lowering (Luganda)	✓	×	×	×
Alternating Meeussen's, lowering (Shona)	×	×	✓	✓

Bounded Meeussen's, lowering (*Luganda*: Niger-Congo): **ISL**

Linguistic generalization: an underlying H lowers to L immediately following another H

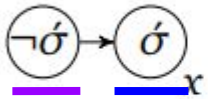
a.	/a-láb-a/	a-láb-a	's/he sees'
b.	/bá-láb-a/	bá-làb-a	'they see'
c.	/bá-lí-láb-a/	bá-lì-làb-a	'they will see'
d.	/a-bá-tá-lí-láb-il-ila/	a-bá-tà-lì-làb-il-ila	'they who will not look after'
e.	/bá-ki-láb-a/	bá-ki-láb-a	'they see it'

- ISL; If we relied on the output, you can see that in **c.** and **d.,** the 3rd TBU would no longer follow an H in the preceding TBU and would not lower.

Bounded Meeussen's, lowering (*Luganda*; Niger-Congo): **ISL**

Linguistic generalization: an underlying H lowers to L immediately following another H

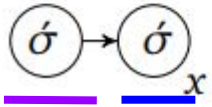
$$\acute{\sigma}_o(x) \stackrel{\text{def}}{=} \acute{\sigma}(x) \wedge \neg\acute{\sigma}(p(x))$$



The corresponding output, *x*, is H iff:

1. **The corresponding input *x* is H,** AND
2. **its predecessor is not H.**

$$\grave{\sigma}_o(x) \stackrel{\text{def}}{=} \grave{\sigma}(x) \vee (\acute{\sigma}(x) \wedge \acute{\sigma}(p(x)))$$



The corresponding output, *x*, is L iff:

1. **The corresponding input, *x*, is L** OR
2. **The corresponding input, *x*, is H** AND **its predecessor is H.**

Bounded Meeussen's, lowering (*Luganda*; Niger-Congo): **ISL**

Linguistic generalization: an underlying H lowers to L immediately following another H

/bá……/
[bá ……]

The corresponding output, *x*, is H iff:
1. The corresponding input *x* is H, AND
2. its predecessor is not H.

/bá – lág – a/
[bá – làb – a]

The corresponding output, *x*, is L iff:
1. The corresponding input, *x*, is L OR
2. The corresponding input, *x*, is H and its predecessor is H.

Survey of tone processes

PROCESS	ISL	A-ISL	RSL	A-RSL
Bounded spread (Bemba)	✓	✓	×	×
Bounded shift (Kuki-Thaadow)	✓	✓	×	×
Unbounded shift to penult (Zigula)	×	✓	×	✓
Unbounded spread to penult (Shambaa)	×	×	×	×
Unbounded Meeussen's, deletion (Arusa)	×	✓	×	×
Bounded Meeussen's, lowering (Luganda)	✓	×	×	×
Alternating Meeussen's, lowering (Shona)	×	×	✓	✓

Alternating Meeussen's, lowering (*Shona*; Niger-Congo): **RSL**

Linguistic generalization: an H is lowered if it follows another H

$$\acute{\sigma}_o(x) \stackrel{\text{def}}{=} \acute{\sigma}(x) \wedge (\#_o(p(x)) \vee \grave{\sigma}_o(p(x)))$$

In prose: **the target output will be a H TBU,** iff

1. **the corresponding input is an H** AND
2. **the predecessor in the corresponding output is a left word edge** OR **a L TBU.**

$$\grave{\sigma}_o(x) \stackrel{\text{def}}{=} \grave{\sigma}(x) \vee (\acute{\sigma}(x) \wedge \acute{\sigma}_o(p(x)))$$

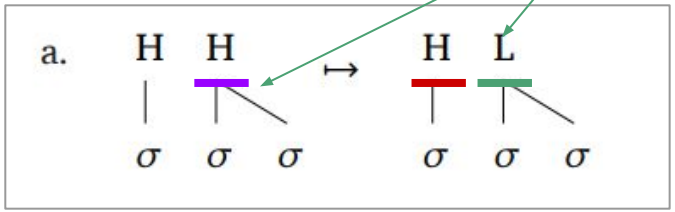
In prose: **the target output will be a L TBU,** iff

1. **the corresponding input is a L** OR
2. **the corresponding input is an H** AND **the predecessor in the corresponding output is a H TBU.**

Alternating Meeussen's, lowering (*Shona*; Niger-Congo): **RSL**

Linguistic generalization: an H is lowered following another H

These are single tones th
correspond to two TBUs!



In prose: **the target output will be a L TBU,** iff

1. **the input is a L** OR
2. **the input is an H** AND **the predecessor in the output is a H TBU.**

Input	Output
/né–hó……/	[né–hò……] (H becomes L)
/né–hó–vé/	[né–hòvè] (H becomes L)

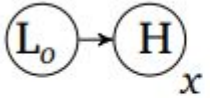
Alternating Meeussen's, lowering (*Shona*; Niger-Congo): **A-RSL**

Linguistic generalization: an H is lowered if it follows another H

1. **The output TBU for *x* is H** iff:

   the corresponding input TBU is an H AND **the predecessor in the output is a L.**

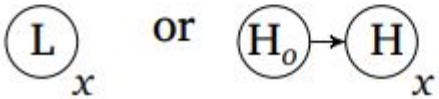
$$H_o(x) \stackrel{\text{def}}{=} H(x) \wedge L_o(p(x))$$



2. **The output TBU for *x* is L** iff:

   A. **the corresponding input TBU is L**
      OR
   B. **the corresponding input TBU is H** AND **its predecessor in the output is H.**

$$L_o(x) \stackrel{\text{def}}{=} L(x) \vee (H(x) \wedge H_o(p(x)))$$



Alternating Meeussen's, lowering (*Shona*; Niger-Congo): **A-RSL**

Linguistic generalization: an H is lowered if it follows another H

1. **The output TBU for *x* is H** iff:

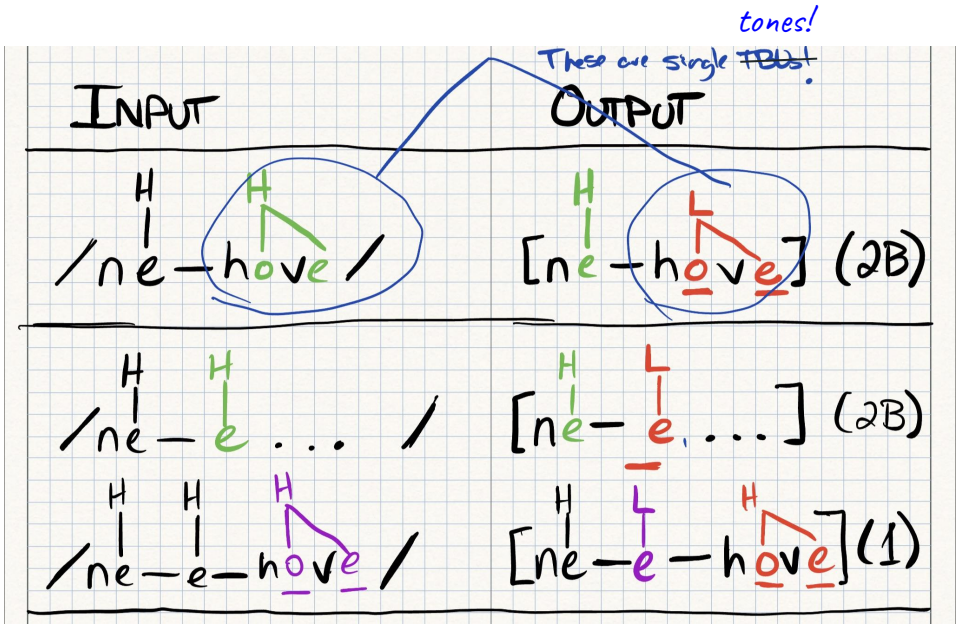
   the corresponding input TBU is an H AND **the predecessor in the output is a L.**

2. **The output TBU for *x* is L** iff:

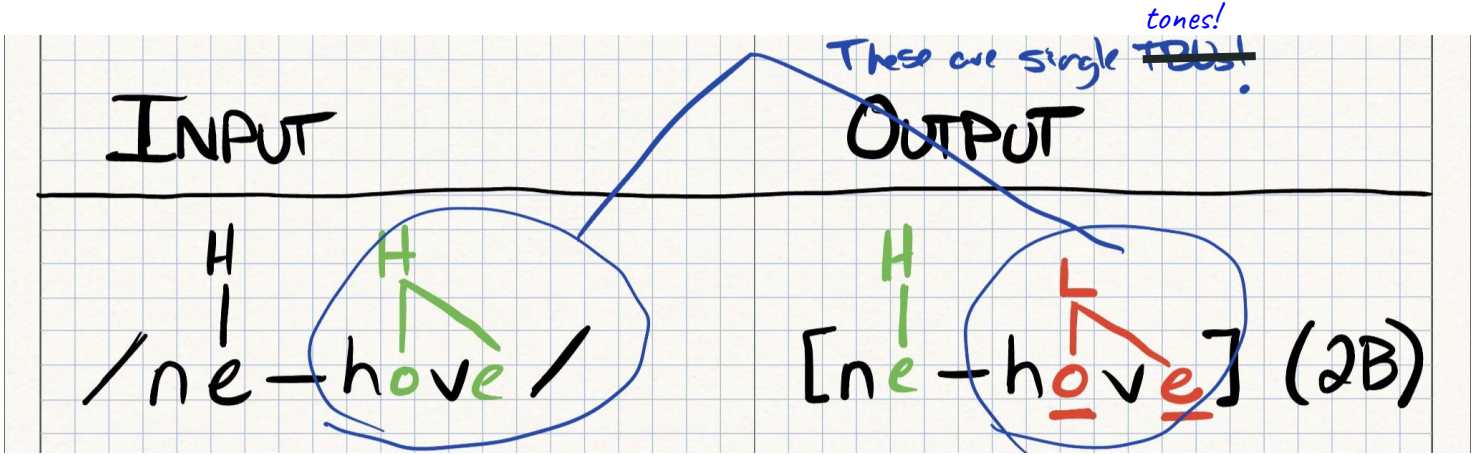
   A. **the corresponding input TBU is L**

   OR

   B. **the corresponding input TBU is H** AND **its predecessor in the output is H.**



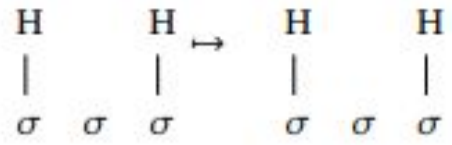
- We have to rely on knowing what the underlying representations of the tones are to get the correct output for all of the derivations in Shona, whether we're doing A-ISL or A-RSL.
- Is this a problem?



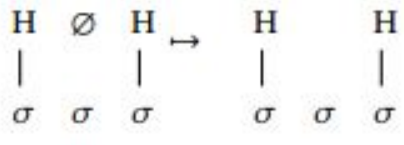
Potential limitation: Having to make representational assumptions w/r/t to what tone is and how to mark it.

Bounded Meeussen's rule in Luganda: a H tone becomes a L tone when it follows a H tone.

- **PROBLEM**: At the level of the tonal tier, there is no way to determine if two tones are adjacent without using a quantifier. So it seems like an A-ISL analysis is not possible.



- **ONE SOLUTION**: Explicitly mark the intervening TBU with a null symbol; while this does make the AR local and allows it to be A-ISL, it is only local given a particular underlying representation.



Potential Limitation: Lookahead & Bidirectionality

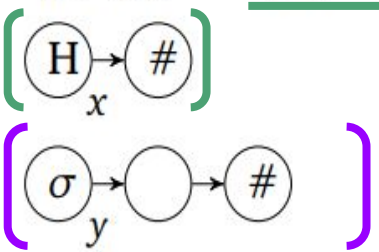
- Tone spreading like in Shambaa do not fit into any of the four classes: is it exceptional?
  - **Generic tone spreading** straightforwardly RSL and A-RSL
  - Shambaa spread to penultimate syllable ⇒ requires “looking ahead” so **not RSL/A-RSL**
- Possible solutions
  - **Input-Output Strictly Local Functions** – allow reference to local information both in input & output
  - **Composition of RSL and ISL functions** – would work for Shambaa and bidirectional processes like **unbounded tone plateauing**
    - **Functional Composition:** one option for combining functions, but maybe too powerful – neither ISL nor OSL are closed under composition
  - **Non-local altogether?**
    - Predictions of locality: e.g. long-distance effects only possible with privative tone

EXTRA SLIDES

Unbounded shift to penult (Zigula; *Niger-Congo*); **A-ISL**

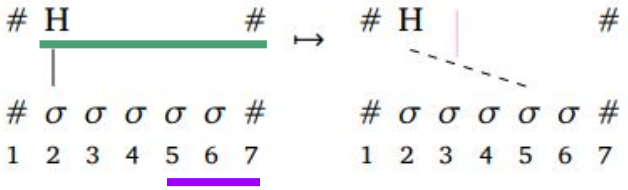
Linguistic generalization: an H shifts to the penultimate TBU.

$$A_o(x,y) \overset{\text{def}}{=} \mathrm{H}(x) \wedge \#(s(x)) \wedge \sigma(y) \wedge \#(s(s(y)))$$



Tonal tier

TBU tier



In prose: Associate the output *x* with the output *y* iff:

1. (There may exist...??) an input *x* on the tonal tier that is H AND is followed by the right edge of the word.

AND

2.(There may exist...??) an input *y* on the TBU tier that is two slots before the right edge of the word.

Survey of tone processes: some trends

PROCESS	ISL	A-ISL	RSL	A-RSL
Bounded spread (Bemba)	✓	✓	×	×
Bounded shift (Kuki-Thaadow)	✓	✓	×	×
Unbounded shift to penult (Zigula)	×	✓	×	✓
Unbounded spread to penult (Shambaa)	×	×	×	×
Unbounded Meeussen's, deletion (Arusa)	×	✓	×	×
Bounded Meeussen's, lowering (Luganda)	✓	×	×	×
Alternating Meeussen's, lowering (Shona)	×	×	✓	✓

- **All bounded processes are either or both ISL and A-ISL.**

Survey of tone processes - Some observations

PROCESS	ISL	A-ISL	RSL	A-RSL
Bounded spread (Bemba)	✓	✓	×	×
Bounded shift (Kuki-Thaadow)	✓	✓	×	×
Unbounded shift to penult (Zigula)	×	✓	×	✓
Unbounded spread to penult (Shambaa)	×	× ?	×	× ?
Unbounded Meeussen's, deletion (Arusa)	×	✓	×	×
Bounded Meeussen's, lowering (Luganda)	✓	×	×	×
Alternating Meeussen's, lowering (Shona)	×	×	✓	✓

- **All unbounded processes are at least auto-segmental**

Survey of tone processes - Some observations

PROCESS	ISL	A-ISL	RSL	A-RSL
Bounded spread (Bemba)	✓	✓	×	×
Bounded shift (Kuki-Thaadow)	✓	✓	×	×
Unbounded shift to penult (Zigula)	×	✓	×	✓
Unbounded spread to penult (Shambaa)	×	×	×	×
Unbounded Meeussen's, deletion (Arusa)	×	✓	×	×
Bounded Meeussen's, lowering (Luganda)	✓	×	×	×
Alternating Meeussen's, lowering (Shona)	×	×	✓	✓

- **The only two types of functions that don't both apply for at least one process are ISL and RSL.**

A computational theory of tone: Remaining questions

- **RECAP**: In Shambaa, H spreads rightwards until it reaches the penult.

/ku-fúmbatiʃ-a/ [ku-fúmbátíʃ-a] ‘to tie securely’

$$\sigma\sigma\sigma\sigma\sigma \mapsto \sigma\sigma\sigma\sigma\sigma$$
$$\sigma\acute{\sigma}\sigma\sigma\sigma \mapsto \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$
$$\sigma\acute{\sigma}\sigma\sigma\sigma\sigma\sigma \mapsto \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$

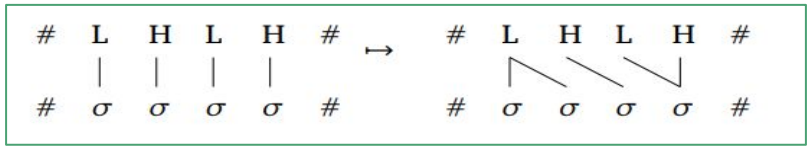
- **PROBLEM**:
  - ***It is not ISL***: Whether the penultimate syllable surfaces as an H depends on a trigger that may be any distance to the left, which is not detectable without a quantifier.
  - ***But it is also not A-ISL***: The need to associate the H to all intervening TBUs between the underlying one and the penult means each target will be progressively further away from the underlying trigger.
  - ***Also not (A-)RSL***: b/c it requires ‘looking ahead’ to determine whether it has reached the penultimate syllable.

- **SOLUTION**: Combine input and output strictly local functions in what they call *Input-output strictly local functions (IOSL)*.

Tone processes cont'd (4/X) - Bounded shift in *Kuki-Thaadow*

/kà zóoŋ lien thúm/ [kà zòoŋ lien thŭm] 'my three big monkeys'

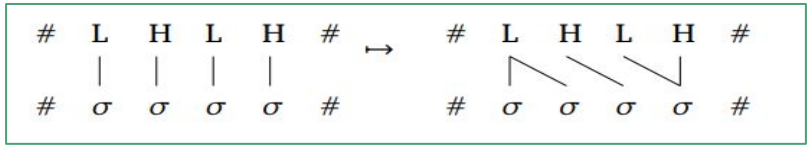
Linguistic generalization: "A string of tones each associate to the following syllable…[while] the first and last tones also remain associated to their underlying TBUs" (13).



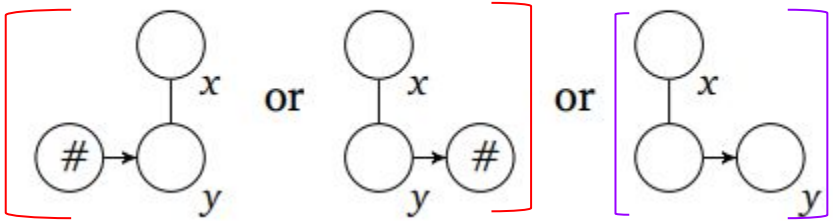
#òóòó# ⟼ #òòóǒ#

Tone processes cont'd (5/X) - Bounded shift in *Kuki-Thaadow;* (A-ISL)

Linguistic generalization: "A string of tones each associate to the following syllable…[while] the first and last tones also remain associated to their underlying TBUs" (13).



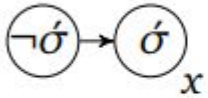
$$A_o(x,y) \stackrel{\text{def}}{=} \big((\#(p(y)) \vee \#(s(y))) \wedge A(x,y)\big) \vee A(x,p(y))$$



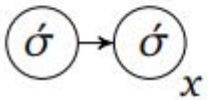
Bounded Meeussen's, lowering (*Luganda*; Niger-Congo): **ISL**

Linguistic generalization: an underlying H lowers to L immediately following another H

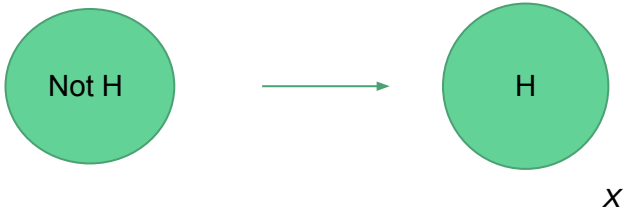
$$\acute{\sigma}_o(x) \stackrel{\text{def}}{=} \acute{\sigma}(x) \wedge \neg\acute{\sigma}(p(x))$$



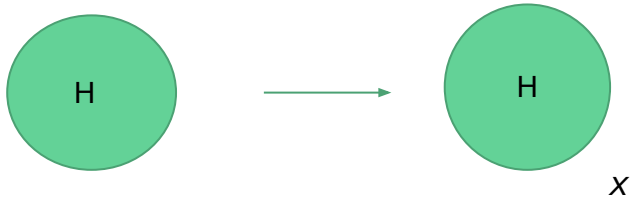
$$\grave{\sigma}_o(x) \stackrel{\text{def}}{=} \grave{\sigma}(x) \vee (\acute{\sigma}(x) \wedge \acute{\sigma}(p(x)))$$



Condition 1: the corresponding output is H, iff the input is:



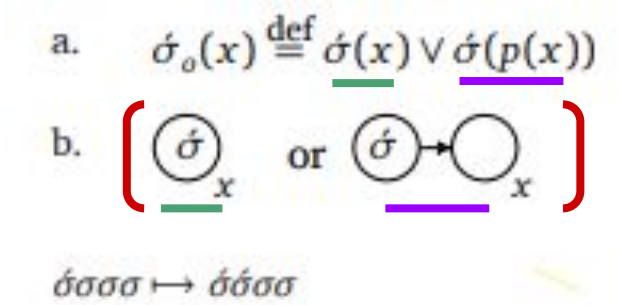
Condition 2: the corresponding output is L, iff the input is:



Bounded spread (Bemba; *Niger-Congo*): **ISL**

Linguistic generalization: an H tone spreads exactly one TBU to the right

Input →



- This is intuitively local.
- The formula is quantifier free (QF) and non-recursive.

Unbounded shift to penult (Zigula; *Niger-Congo*); **A-ISL**

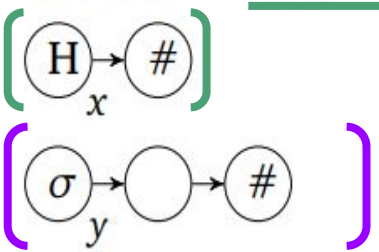
Linguistic generalization: an H shifts to the penultimate TBU.

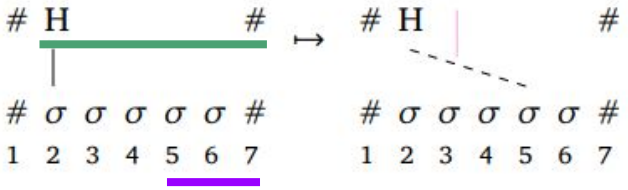
/ku-songoloz-a/	[ku-songoloz-a]	‘to avoid’
/á-songoloz-a/	[a-songolóz-a]	‘he/she is avoiding’
/ku-lómbez-a/	[ku-lombéz-a]	‘to ask’
/ku-lómbez-ez-a/	[ku-lombez-éz-a]	‘to ask for’
/ku-lómbez-ez-an-a/	[ku-lombez-ez-án-a]	‘to ask for each other’

Unbounded shift to penult (Zigula; *Niger-Congo*); **A-ISL**

Linguistic generalization: an H shifts to the penultimate TBU.

$$A_o(x,y) \overset{\text{def}}{=} \mathrm{H}(x) \wedge \#(s(x)) \wedge \sigma(y) \wedge \#(s(s(y)))$$





- Note that the formula above does not depend on there actually being an H in the input…It only dictates that an *x* and a *y* that satisfy it are associated. If no such *x* and *y* exist–i.e., if there are no H tones–then no association takes place." (p. 15).
- This is why an existential quantifier, *E,* is not needed.

Unbounded spread to penult (*Shambaa*; Niger-Congo)

Linguistic generalization: H spreads rightward until it reaches the penult

/ku-hand-a/	[ku-hand-a]	‘to plant’
/ku-fúmbatiʃ-a/	[ku-fúmbátíʃ-a]	‘to tie securely’
/ku-hand-ij-an-a/	[ku-hand-ij-an-a]	‘to plant for each other’
/ku-fúmbatiʃ-ij-an-a/	[ku-fúmbátíʃ-íj-án-a]	‘to tie securely for each other’
/ku-ʃuntʰ-a/	[ku-ʃuntʰ-a]	‘to wash’
/ku-tʃí-ʃuntʰ-a/	[ku-tʃí-ʃúntʰ-a]	‘to wash’
/ku-ɣoʃo-a-ɣoʃo-a/	[ku-ɣoʃo-a-ɣoʃo-a]	‘to do repeatedly’
/ku-tʃí-ɣoʃo-a-ɣoʃo-a/	[ku-tʃí-ɣóʃó-á-ɣóʃó-a]	‘to do repeatedly’

Unbounded spread to penult (*Shambaa*; Niger-Congo)

Linguistic generalization: H spreads rightward until it reaches the penult

$$\sigma\sigma\sigma\sigma\sigma \mapsto \sigma\sigma\sigma\sigma\sigma$$
$$\sigma\acute{\sigma}\sigma\sigma\sigma \mapsto \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$
$$\sigma\acute{\sigma}\sigma\sigma\sigma\sigma\sigma \mapsto \sigma\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\acute{\sigma}\sigma$$

- Unbounded spread is not ISL; whether the penultimate syllable surfaces as toneless or H depends on a trigger that may be any distance to the left, which is not detectable without a quantifier.
- Not A-ISL
- Not RSL
- Not A-RSL

ISL Functions: Local vs. Long Distance

- **ISL Functions = precise & computational notion** of what it means to be local
- **Kikongo:** suffix *-idi* surfaces as *-ini* when it attaches to a stem **containing a nasal**
  - Embedded predecessor functions: **no upper bound on how many preceding segments** ❌
  - Would **require a quantifier** to examine the entire stem (i.e. $\exists (x) \wedge \text{nasal}(x)$)
    ∴non-local & not ISL