Jardin et al. 2014

LIN 629, Fall 2022

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Introduction

1. They present the Structured Onward Subsequential Function Inference Algorithm (SOSFIA) which can identify some classes of sequential functions in linear time and data.
2. The main way they do this is by using *a priori* knowledge of structure shared by functions in the class (No state merging!).
3. One of these classes is are the Input Strictly Local functions.

Preliminaries

Prefixes

The prefixes of $w \in \Sigma^*$ are,

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Examples

$$\texttt{pref}(\lambda) = \{\lambda\}$$
$$\texttt{pref}(\mathsf{aba}) = \{\lambda, \mathsf{a}, \mathsf{ab}, \mathsf{aba}\}$$
$$\texttt{pref}(\{\mathsf{aba}, \mathsf{bb}\}) = \{\lambda, \mathsf{a}, \mathsf{ab}, \mathsf{abab}, \mathsf{bb}\}$$

Preliminaries

Shared Prefixes

The *shared prefixes* of S are the prefixes shared by all strings in S.

$$\mathtt{sh\_pref}(S) = \bigcap_{w \in S} \mathtt{pref}(w)$$

Examples

$$\begin{aligned}
\mathtt{sh_pref}(\{\ \}) &= \{\ \} \\
\mathtt{sh_pref}(\{\mathsf{aba}, \mathsf{bb}\}) &= \{\lambda, \mathsf{a}, \mathsf{ab}, \mathsf{aba}\} \cap \{\lambda, \mathsf{b}, \mathsf{bb}\} \\
&= \{\lambda\} \\
\mathtt{sh_pref}(\{\mathsf{aba}, \mathsf{a}\}) &= \{\lambda, \mathsf{a}, \mathsf{ab}, \mathsf{aba}\} \cap \{\lambda, \mathsf{a}\} \\
&= \{\lambda, \mathsf{a}\}
\end{aligned}$$

Preliminaries

Longest Common Prefix

The longest common prefix ($\texttt{lcp}$) of S is then,

$$\texttt{lcp}(S) = \{w \in \texttt{sh\_pref}(S) \mid \forall v \in \texttt{sh\_pref}(S) : |v| \leq |w|\}$$

We define the $\texttt{lcp}$ of the empty set to be λ.

Examples

$$\texttt{lcp}(\{\ \}) = \lambda$$
$$\texttt{lcp}(\{\mathsf{aba}, \mathsf{a}\}) = \mathsf{a}$$

Preliminaries

Relations

Given an input alphabet Σ and an output alphabet Δ, a relation from Σ to Δ is a subset of $\Sigma^* \times \Delta^*$.

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We call $t : \Sigma^* \rightarrow \Delta^*$ a function iff $\forall w \in \Sigma^*$,

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Totality

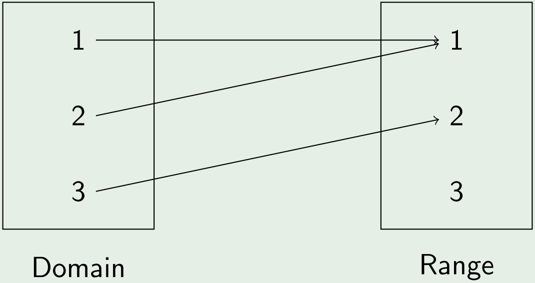
A function $t : \Sigma^* \rightarrow \Delta^*$ is total iff,

$$\forall w \in \Sigma^*, \exists v \in \Delta^* \text{ s.t. } (w, v) \in t$$

Otherwise, the function is partial.

Preliminaries

Examples



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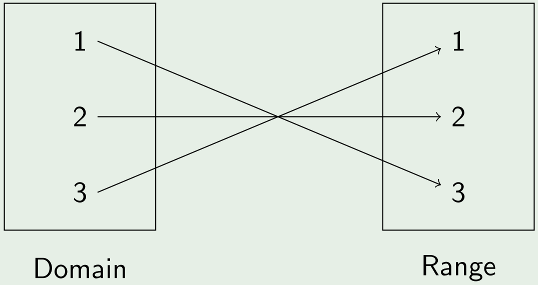

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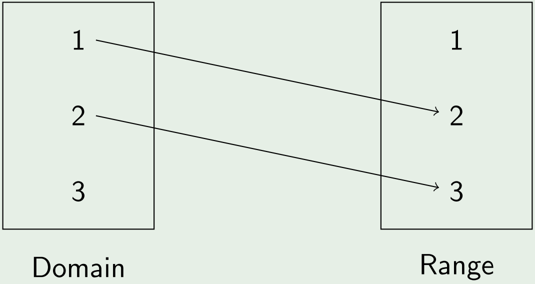

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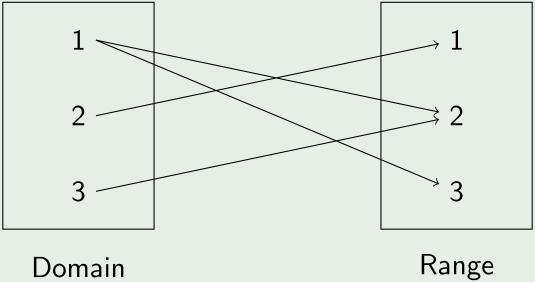

Preliminaries

Tails of a Function

For any function $t : \Sigma^* \rightarrow \Delta^*$ and $w \in \Sigma^*$, we define the tails of w with respect to t as,

$$\texttt{tails}_t(w) = \{(x, v) \mid t(wx) = uv \wedge u = \texttt{lcp}(t(w\Sigma^*))\}$$

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Examples

Let $\Sigma = \Delta = \{\mathrm{a}, \mathrm{b}, \mathrm{p}\}$ and t be the identity function.

$$\begin{aligned}\texttt{tails}_t(\mathrm{ba}) &= \{(\mathrm{a}, \mathrm{a}), (\mathrm{b}, \mathrm{b}), (\mathrm{p}, \mathrm{p}), (\mathrm{ba}, \mathrm{ba}), \cdots\} \\ \texttt{tails}_t(\mathrm{bab}) &= \{(\mathrm{a}, \mathrm{a}), (\mathrm{b}, \mathrm{b}), (\mathrm{p}, \mathrm{p}), (\mathrm{ba}, \mathrm{ba}), \cdots\}\end{aligned}$$

Note that $\texttt{tails}_t(\mathrm{ba}) = \texttt{tails}_t(\mathrm{bab})$.

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Examples

Let $\Sigma = \Delta = \{\text{a}, \text{b}, \text{p}\}$ and t require a "b" to follow every "a".

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Preliminaries

Definition

Two strings $w, w' \in \Sigma^*$ are tail-equivalent with respect to a function t iff,

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which we denote as $w \sim_t w'$.

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The equivalence relation $\sim_t$ partitions Σ^*.

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How many blocks does $\sim_t$ partition Σ^* into?

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A function t is subsequential iff $\sim_t$ partitions Σ^* into *finitely* many blocks.

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Examples

Let $\Sigma = \Delta = \{\text{a}, \text{b}, \text{p}\}$ and t be the identity function.

Is t a subsequential function?

Let $\Sigma = \Delta = \{\text{a}, \text{b}, \text{p}\}$ and f require a "b" to follow every "a".

Is f a subsequential function?

Traditional Subsequential Transducers

Definition

A subsequential finite-state transducer (SFST) is a 6-tuple $\langle Q, q_0, \Sigma, \Delta, \delta, p \rangle$ where,

1. Q is a *finite* set of states.
2. $q_0 \in Q$ is the initial state.
3. Σ is a *finite* input alphabet.
4. Δ is a *finite* output alphabet.
5. $\delta \subseteq Q \times \Sigma \times \Delta^* \times Q$ is the transition function.
6. $p : Q \to \Delta^*$ is the output function.

Additionally, the transition function must be deterministic. That is,

$$(q, \sigma, w, r), (q, \sigma, v, s) \in \delta \implies (r = s) \wedge (w = v)$$

Traditional Subsequential Transducers

Theorem

If we denote the relation described by a SFST τ as,

$$R(\tau) = \{(w, vv') \mid (\exists q)[(q_0, w, v, q) \in \delta^* \wedge p(q) = v']\}$$

Then for every for every subsequential function t, there is a subsequential transducer τ which computes it ($R(\tau) = t$), and likewise for every subsequential transducer τ, $R(\tau)$ is a subsequential function.

Delimited Subsequential Transducers

Definition

A delimited subsequential finite-state transducer (DSFST) is a 6-tuple $\langle Q, q_0, q_f, \Sigma, \Delta, \delta \rangle$ where,

1. Q is a *finite* set of states.
2. $q_0 \in Q$ is the initial state.
3. $q_f \in Q$ is the final state.
4. Σ is a *finite* input alphabet.
5. Δ is a *finite* output alphabet.
6. $\delta \subseteq Q \times \Sigma \cup \{\ltimes, \rtimes\} \times \Delta^* \times Q$ is the transition function.

Where $\ltimes, \rtimes \notin \Sigma$ and the following holds,

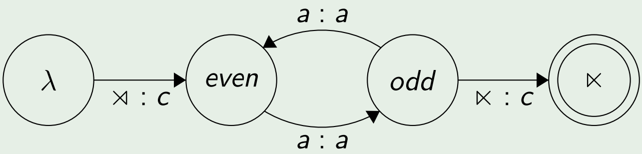
1. $(q, \sigma, u, q') \in \delta \implies (q \neq q_f) \wedge (q' \neq q_0)$,
2. $(q, \sigma, u, q_f) \in \delta \implies (\sigma = \ltimes) \wedge (q \neq q_0)$,
3. $[(q_0, \sigma, u, q') \in \delta \implies \sigma = \rtimes] \wedge [(q_0, \rtimes, u, q') \implies q = q_0]$
4. $(q, \sigma, w, r), (q, \sigma, v, s) \in \delta \implies (r = s) \wedge (w = v)$

Reachable States

Definition

A state q in a DSFST τ is reachable iff there exists $w \in \Sigma^*, v \in \Delta^*$ such that $(q_0, \rtimes w, v, q) \in \delta^*$.

Examples



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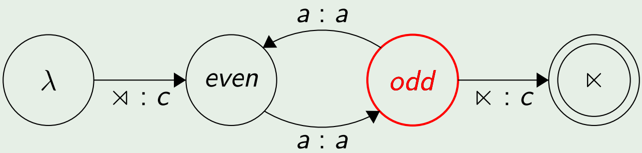

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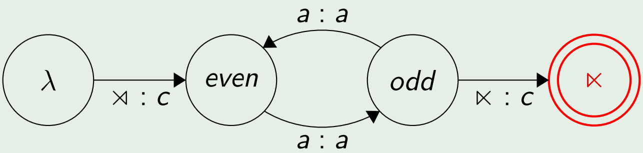

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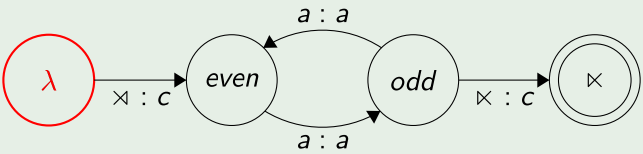
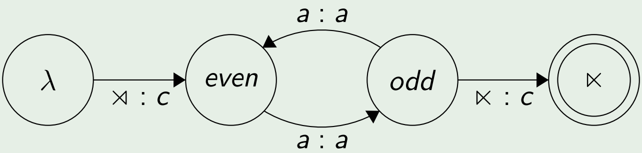


Useful States

Definition

A state q in a DSFST τ is useful iff q is *reachable* and there exists $w \in \Sigma^*, v \in \Delta^*$ such that $(q, w\ltimes, v, q_f) \in \delta^*$.

Examples



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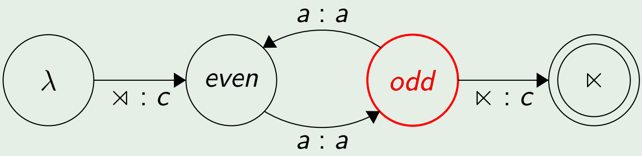

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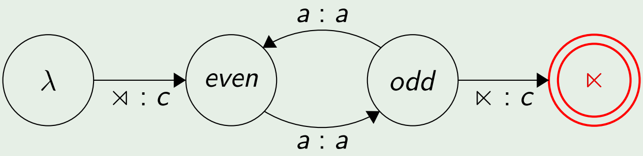

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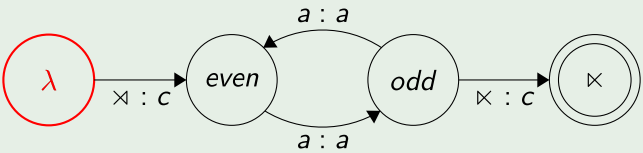

Trimmed Transducers

Definition

A transducer is trimmed iff every state other than q_0 and q_f is *useful*.

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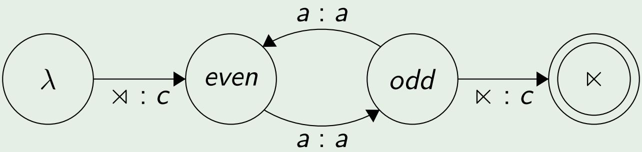

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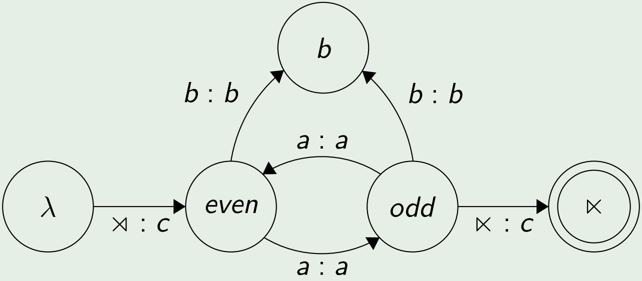

Onward Transducers

Definition

A DSFST τ is onward iff the outgoing transitions of each noninitial state share no nonempty prefix.

$$(\forall q \in (Q - q_0))[\texttt{lcp}\{w \in \Sigma^* \mid (\exists a \in \Sigma, r \in Q)[(q, a, w, r) \in \delta]\} = \lambda]$$

Intuitively, a transducer is *onward* iff there is no delay in writing the output strings. As the input symbols are consumed, the output is written the moment it is determined.

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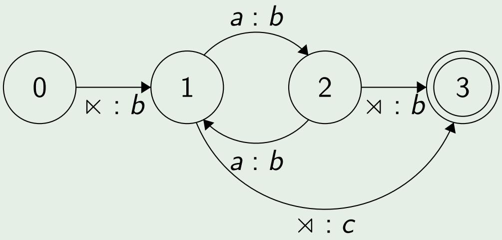

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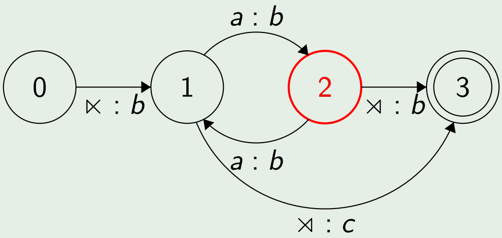

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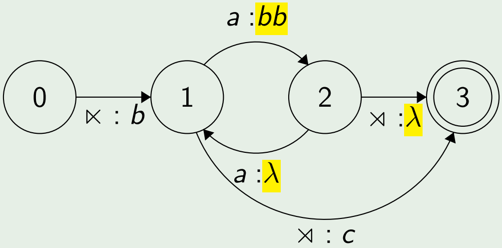

Deriving Onward Transducers

Theorem

For any trimmed DSFST τ, there is an onward trimmed DSFST τ' such that $R(\tau) = R(\tau')$

Remarks

What this is saying is that for any DSFST τ, there is an *onward DSFST* with the exact same structure which recognizes the same function described by τ.

For the purposes of this presentation I state this without proof. The important thing is that there is some canonical onward DSFST for every DSFST and additionally there is a constructive way to find it.

Learning Paradigm

The results for the algorithm presented in this paper (SOSFIA) are demonstrated in the identification in the limit learning paradigm.

Definition

A class $\mathbb{T}$ of functions is represented by a class $\mathbb{R}$ of representations if every $r \in \mathbb{R}$ is of finite size and there is a function $\mathcal{L} : \mathbb{R} \rightarrow \mathbb{T}$ which is both total and surjective

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Remark

The class of subsequential functions can be represented by the class of DSFSTs.

Learning Paradigm

Definition

Let $\mathbb{T}$ be a class of functions represented by some class $\mathbb{R}$ of representations.

1. A sample S for a function $t \in \mathbb{T}$ is a finite set of data consistent with t, that is to say,
$$(w, v) \in S \iff t(w) = v$$
2. The size of a sample S is the sum of the length of the strings it is composed of.
$$|S| = \sum_{(w,v)\in S} |w| + |v|$$
3. A $(\mathbb{T}, \mathbb{R})$-learning algorithm $\mathfrak{A}$ is a program that takes as input a sample for a function $t \in \mathbb{T}$ and outputs a representation from $\mathbb{R}$.

Learning Paradigm

Definition (Characteristic Sample)

For a $(\mathbb{T}, \mathbb{R})$-learning algorithm $\mathfrak{A}$, a sample CS is a characteristic sample of a function $t \in \mathbb{T}$ if for all samples S such that $CS \subseteq S$, $\mathfrak{A}$ returns a representation r such that, for all $w \in \mathtt{dom}(t)$, $r(w) = t(w)$.

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In English, this is saying that a sample is a characteristic sample if adding more elements does not (loosely speaking) change the output for $\mathfrak{A}$.

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This is the definition Jardine et al. 2014 uses so we will stick to that.

Output-Empty Transducers

Definition

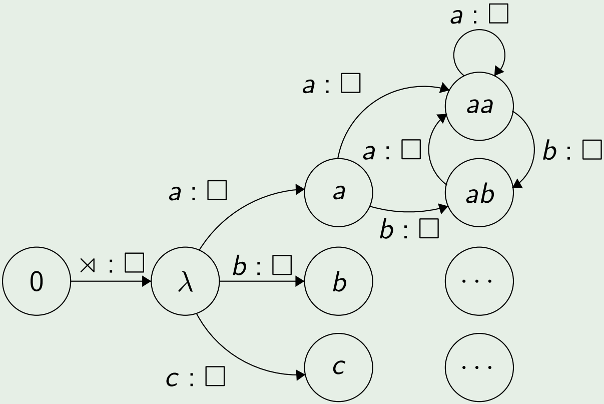
An output-empty transducer $\tau_{\Box}$ is a DSFST $\langle Q, q_0, q_f, \Sigma, \{\Box\}, \delta \rangle$ such that for all $(q, a, u, q') \in \delta$, $u = \Box$.

An output-empty transducer τ defines a class of functions $\mathcal{T}$ which is exactly the set of functions which can be created by taking the states and transitions of τ and replacing the blanks with output strings, maintaining onwardness.

The goal is to create an algorithm that can fill in these blanks.

Output-Empty Transducers

Example



The Inference Algorithm

The algorithm takes an output-empty transducer $\tau_{\Box}$, and a finite sample S from some target function $t \in \mathcal{T}_{\tau_{\Box}}$.
At each state, it sets the output of each outgoing transition to be *the minimal change in the output* generated by this transition according to S.

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Definition

The common output of an input prefix w in a sample $S \subset \Sigma^* \times \Delta^*$ for t is the `lcp` of all $t(wv)$ that are in S.

$$\texttt{common\_out}_S(w) = \texttt{lcp}(\{u \in \Delta^* \mid \exists v \text{ s.t. } (wv, u) \in S\})$$

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$$\texttt{min\_change}_S(\sigma, w) = \begin{cases} \texttt{common\_out}_S(\sigma) & \textit{if } w = \lambda \\ \texttt{common\_out}_S(w)^{-1}\texttt{common\_out}_S(w\sigma) & \textit{otherwise} \end{cases}$$

common_out

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Example

Consider the following sample,

$$S = \left\{ \begin{array}{lr} (anpa, ama), & (anpo, amo) \\ (ana, ana), & (ano, ano) \\ (anda, anda), & (ando, ando) \end{array} \right\}$$

$\texttt{common\_out}_S(a) = ???$

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Example

Consider the following sample,

$$S = \left\{ \begin{array}{ll} (\mathbf{a}npa, ama), & (\mathbf{a}npo, amo) \\ (\mathbf{a}na, ana), & (\mathbf{a}no, ano) \\ (\mathbf{a}nda, anda), & (\mathbf{a}ndo, ando) \end{array} \right\}$$

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$$\begin{aligned} \texttt{common\_out}_S(an) &= \texttt{lcp}(\{ama, amo, ana, ano, anda, ando\}) \\ &= a \end{aligned}$$

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Definition

The minimal change in the output in $S \subset \Sigma^* \times \Delta^*$ from w to $w\sigma$ is,

$$\texttt{min\_change}_S(\sigma, w) = \begin{cases} \texttt{common\_out}_S(\sigma) & \textit{if } w = \lambda \\ \texttt{common\_out}_S(w)^{-1}\texttt{common\_out}_S(w\sigma) & \textit{otherwise} \end{cases}$$

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Example

Consider the following sample,

$$S = \begin{Bmatrix} (anpa, ama), & (anpo, amo) \\ (ana, ana), & (ano, ano) \\ (anda, anda), & (ando, ando) \end{Bmatrix}$$

$\texttt{min\_change}_S(n, a) = \texttt{common\_out}_S(a)^{-1}\texttt{common\_out}_S(an)$

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Example

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Continued by Salam