Regular Expressions

Syntax

REs include

- each $\sigma \in \Sigma$
- ϵ
- $\varnothing$

If R and S are REs then so are

- $(\mathrm{R}\cdot\mathrm{S})$ *(concatenation)*
- $(\mathrm{R}+\mathrm{S})$ *(union)*
- (R^*) *(Kleene star)*

Semantics

- $[\![\sigma]\!] = \{\sigma\}$
- $[\![\epsilon]\!] = \{\epsilon\}$
- $[\![\varnothing]\!] = \{\}$

- $[\![(\mathrm{R}\cdot\mathrm{S})]\!] = [\![\mathrm{R}]\!] \cdot [\![\mathrm{S}]\!]$
- $[\![(\mathrm{R}+\mathrm{S})]\!] = [\![\mathrm{R}]\!] \cup [\![\mathrm{S}]\!]$
- $[\![(\mathrm{R}^*)]\!] = [\![\mathrm{R}]\!]^*$

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Generalized Regular Expressions

Syntax

GREs include

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- ϵ
- $\varnothing$

If R and S are GREs then so are

- $(\mathrm{R}\cdot\mathrm{S})$ *(concatenation)*
- $(\mathrm{R}+\mathrm{S})$ *(union)*
- (R^*) *(Kleene star)*
- $(\mathrm{R}\&\mathrm{S})$ *(intersection)*
- $(\overline{\mathrm{R}})$ *(complement)*

Semantics

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- $[\![(\mathrm{R}^*)]\!] = [\![\mathrm{R}]\!]^*$
- $[\![(\mathrm{R}\&\mathrm{S})]\!] = [\![\mathrm{R}]\!] \cap [\![\mathrm{S}]\!]$
- $[\![\overline{\mathrm{R}}]\!] = \Sigma^* - [\![\mathrm{R}]\!]$

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Generalized Regular Expressions

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Adding intersection and complement does not increase power of REs!

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Cat-Union Expressions

Syntax

CUEs include

- each $\sigma \in \Sigma$
- ϵ
- $\varnothing$

If R and S are CUEs then so are

- $(\mathrm{R}\cdot\mathrm{S})$ *(concatenation)*
- $(\mathrm{R}+\mathrm{S})$ *(union)*
- (R^*) *(Kleene star)*
- $(\mathrm{R}\&\mathrm{S})$ *(intersection)*
- $(\overline{\mathrm{R}})$ *(complement)*

Semantics

- $[\![\sigma]\!] = \{\sigma\}$
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- $[\![(\mathrm{R}\&\mathrm{S})]\!] = [\![\mathrm{R}]\!] \cap [\![\mathrm{S}]\!]$
- $[\![\overline{\mathrm{R}}]\!] = \Sigma^* - [\![\mathrm{R}]\!]$

Cat-Union Expressions

Syntax

CUEs include

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If R and S are CUEs then so are

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- $[\![(\mathrm{R}\&\mathrm{S})]\!] = [\![\mathrm{R}]\!] \cap [\![\mathrm{S}]\!]$
- $[\![\overline{\mathrm{R}}]\!] = \Sigma^* - [\![\mathrm{R}]\!]$

Theorem: $[\![\mathrm{CUE}]\!] = \{L \subseteq \Sigma^* \mid |L| \text{ is finite}\} \subsetneq [\![\mathrm{RE}]\!] = [\![\mathrm{GRE}]\!]$

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Star-Free Regular Expressions

Syntax

SFEs include

- each $\sigma \in \Sigma$
- ϵ
- $\varnothing$

If R and S are SFEs then so are

- $(\mathrm{R}\cdot\mathrm{S})$ *(concatenation)*
- $(\mathrm{R}+\mathrm{S})$ *(union)*
- (R^*) *(Kleene star)*
- $(\mathrm{R}\&\mathrm{S})$ *(intersection)*
- $(\overline{\mathrm{R}})$ *(complement)*

Semantics

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Star-Free Regular Expressions

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Theorem: $[\![\mathrm{SFE}]\!] \subsetneq [\![\mathrm{RE}]\!] = [\![\mathrm{GRE}]\!]$

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Expression Summary

Finite Languages	Star-Free Languages	Regular Languages
concatenation	concatenation	concatenation
union	union	union
	complement	**Kleene star**
	(intersection)	(complement)
		(intersection)

Expressivity $\longrightarrow$

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